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# DEPOT LOCATION

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*ABSTRACT:*

*This paper considers the distribution of a number of commodities from their place of production to a large number of demand points. The factors which influence the creation of intermediate depots between supply and demand are discussed. Several models of spatial dispersion are considered. The first model assumes a continuous model for the demand and shows the relationship between the number of depots, the fixed cost of a depot, and the transport cost. The second model is an integer programming model in which the demand sites are a discrete set of points. This model has been used to study the distribution of bread and milk to the home delivery market in the Adelaide Metropolitan area. Results of this study are given.*

## INTRODUCTION

The location of plants and depots (warehouses) to distribute goods to a market has received considerable attention in the literature. Generally the approaches can be classified as continuous models in which depots can be placed anywhere or discrete models in which depots may be placed at a finite number of points. The work reported in this paper is an application of the discrete method to the distribution of commodities to the home market in metropolitan Adelaide. We attempt to evaluate cost savings if commodities, which are currently being delivered separately, use a common delivery system. The aim of the study has been the determination of the number of depots which are built in an optimum distribution pattern for various values of the system parameters.

Bos (1965) and Geoffrion (1976) have discussed how continuous models can be used to estimate the approximate number of depots needed in a region from the fixed cost of the depot and the unit transport cost of the goods from the depot to the demand. One result is that if the demand for a commodity is changed by a factor of $\rho$ the number of depots increases by $\rho^{2/3}$. Clearly there are economies in combining distribution systems if the only effect is to increase volume in a region and there are no side effects. Doubling the volume or combining two systems of equal volume implies a multiplier of 1.58 for depots.

The above analysis provides good approximations but generally discrete models using integer programming methods have been used in applications to particular situations. Kaufman, Eede and Hansen (1977) give a number of references to the literature on the discrete approach to the depot location problem. The formulation considered in this paper differs from standard formulations in that several commodities are present and limitations are placed on the resources available at each supply to ensure that each plant maintains its current share of the market. We allow the possibility of direct delivery from plant to demand so that in effect each plant may act as a depot. However, a plant only produces one commodity so that a plant cannot satisfy demand for several commodities.

The important parameters in the system are the fixed cost of a depot, the variable handling cost at a depot, and the transport costs. The three transport costs considered are plant to demand, plant to depot, and depot to demand. The system has competition between the cost of direct delivery from the plant to the demand versus the cost of indirect delivery via a depot in which fixed and variable handling costs contribute. The output of the model gives the optimum distribution pattern and in particular the number of depots to be built and the percentage of the flow which utilises the depots.

DATA

This paper considers the distribution of commodities to the home market in the Adelaide metropolitan region. Typical commodities are bread, milk, and soft drinks. We consider 12 plants of which 8 produce only the first commodity and 4 produce only the second commodity. The location of the plants is fixed and the available resources of the plants is known. We consider the weekly distribution to smooth daily fluctuations.

The possible sites for location of depots are spaced uniformly over the populated region of the Adelaide plains. The number of sites totals 25. Demand figures were obtained by assuming that demand at the household level is proportional to the number of children under 15 years of age. The 1976 census subdivisions were used as the basis of estimating demand. There are 143 such subdivisions. Estimates of the centroids of the subdivisions were made.

Transportation cost estimates have been based upon using the Manhattan distance between two points $(x_1,y_1)$ and $(x_2,y_2)$ defined by

$d_{12} = |x_1 - x_2| + |y_1 - y_2|$ . Adelaide is a very flat region with few natural barriers to the free movement of goods. The assumption of the above distance as proportional to travel cost is a reasonable approximation in this case. The above distance is the "access" distance to a subdivision and does not take into account the distance involved in making the deliveries (routing) within the subdivision. A previous study by Lee and Mills (1975) has shown that often the routing cost is greater than the access cost and has significant improvement potential when compaction of a route takes place.

The above factors have been taken into account in selecting the per unit per mile transportation cost parameters $t_{ij}$ between plant and depot, $t_{jk}$ between depot and demand, and $t_{ik}$ between plant and demand. Generally, $t_{jk} > t_{ij}$ because smaller vehicles are used on delivery to the homes and "routing" costs are involved. Also $t_{ik} > t_{jk}$ since only one commodity can be delivered by plant to demand deliveries so that economies due to compaction do not take place.

We assume that the cost of operating a depot is a concave function of the throughput or volume. Elshafei (1975) has compared this model with several other models.

We assume the depot cost has a fixed cost component and a variable cost which is linear with volume. The model has been run for a range of values of these costs.

## MODEL

The depot location problem studied here may be formulated as follows: given a set of plants, a set of locations where depots may be built and a known demand from a given set of customers which must be satisfied, determine the number and location of depots to be established in order to minimise total distribution costs. This problem can be expressed as the following program when there are two commodities to be distributed. The extension to more commodities is trivial.

$$\text{Minimise} \quad \sum_{ij} a_{ij} S_i^{(1)} W_{ij} + \sum_{ij} b_{ij} S_i^{(2)} X_{ij}$$

$$+ \sum_{jk} c_{jk} D_k^{(1)} Y_{jk} + \sum_{jk} d_{jk} D_k^{(2)} Z_{jk}$$

$$+ \sum_{ik} g_{ik} D_k^{(1)} P_{ik} + \sum_{ik} h_{ik} D_k^{(2)} r_{ik}$$

$$+ \sum_j f_j v_j + \sum_j e_j u_j$$

subject to

$$\sum_j W_{ij} + \frac{1}{S_i^{(1)}} \sum_k D_k^{(1)} P_{ik} \leq 1 \quad \text{all } i \qquad (1)$$

$$\sum_j X_{ij} + \frac{1}{S_i^{(2)}} \sum_k D_k^{(2)} R_{ik} \leq 1 \quad \text{all } i \qquad (2)$$

$$\sum_j Y_{jk} + \sum_i P_{ik} = 1 \quad \text{all } k \qquad (3)$$

$$\sum_j Z_{jk} + \sum_i R_{ik} = 1 \quad \text{all } k \qquad (4)$$

$$\sum_i S_i^{(1)} W_{ij} - \sum_k D_k^{(1)} Y_{jk} = 0 \quad \text{all } j \qquad (5)$$

$$\sum_i S_i^{(2)} X_{ij} - \sum_k D_k^{(2)} Z_{jk} = 0 \quad \text{all } j \qquad (6)$$

$$\sum_i S_i^{(2)} X_{ij} - \sum_k D_k^{(2)} Z_{jk} = 0 \quad \text{all } j \qquad (6)$$

$$\sum_i S_i^{(1)} W_{ij} + \sum_k S_k^{(2)} X_{ij} - u_j = 0 \quad \text{all } j \qquad (7)$$

$$u_j - M\, v_j < 0 \qquad \text{all } j \qquad (8)$$

Here

$W_{ij}$ = fraction of commodity 1 at supply i sent to depot j

$X_{ij}$ = fraction of commodity 2 at supply i sent to depot j

$Y_{ij}$ = fraction of commodity 1 at demand k sent from depot j

$Z_{jk}$ = fraction of commodity 2 at demand k sent from depot j

$P_{ik}$ = fraction of commodity 1 at demand k sent from supply i

$R_{ik}$ = fraction of commodity 2 at demand k sent from supply i

$u_j$ = throughput of depot j

$v_j$ = boolean variable taking value of 1 if depot j is built and 0 otherwise.

The coefficients $a_{ij}$, $b_{ij}$, $c_{jk}$, $d_{jk}$, $g_{ik}$, $h_{ik}$ are per unit transport cost coefficients, $f_j$ and $e_j$ are the fixed and variable costs associated with a depot, while $S_i$ and $D_k$ are the supply resources and demand requirements.

Constraints (1) and (2) represent limitations in the available supply of the commodities at the various plants. Constraints (3) and (4) represent the requirement that demands must be fulfilled. Constraints (5) and (6) represent conservation of commodity flow at the intermediate depots. We have assumed that the goods are perishable so that no intermediate storage of commodities is allowed. Constraints (7) and (8) calculate the throughput of a depot and ensure a depot with positive throughput pays the fixed cost of building the depot. These later constraints are the only ones involving the different commodities. Without those constraints the problem would decompose into 2 separate problems.

Other formulations of the depot location problem are possible but the above allows multicommodities and limitations on the supplies which we found useful for our problem. There are a large number of variables in the above formulation. For I plants, J depots and K demands there are 2(I + 2J + K) constraints and 2(I.J + J.K + I.K) + 2J variables of which J are 0.1 integer variables. For a problem with 12 plants, 25 depots and 140 demands we have 11,010 variables. This number can be reduced slightly since not all plants produce all commodities. Another way of reducing the variables is to assume that a depot gets its delivery from the closest supply and eliminating all other variable representing supply to the depot. Unfortunately the resource limitations on a supply can lead to infeasibility in this situation.

## COMPUTATIONAL EXPERIENCE

Results have been obtained for different combinations for the parameters. The relationship between the objective function coefficients and the transport parameters is

$$a_{ij} = b_{ij} = t_{ij}d_{ij}$$

$$c_{jk} = d_{jk} = t_{jk}d_{jk}$$

$$g_{ik} = h_{ij} = t_{ij}d_{ij}$$

In table 1 some typical results are given for the case of 12 plants, 12 possible depots and demand lumped into 37 regions.

Table 1. Input Parameters and Results

| f | u | $t_{ij}$ | $t_{jk}$ | $t_{ik}$ | Z | j | % |
|---|---|---|---|---|---|---|---|
| 150 | .04 | .001 | .005 | .01 | 78554 | 9 | 44 |
| 450 | .03 | .001 | .005 | .01 | 73100 | 9 | 49 |
| 150 | .03 | .001 | .005 | .01 | 70700 | 10 | 49 |
| 1.5 | .03 | .001 | .005 | .01 | 69232 | 11 | 51 |
| 150 | .03 | .001 | .005 | .008 | 64930 | 8 | 43 |
| 150 | .03 | .001 | .005 | .006 | 57321 | 5 | 31 |
| 150 | .03 | .001 | .005 | .005 | 52587 | 4 | 25 |

Here Z is the value of the objective function, j is the number of depots built and % is the percentage of the flow which uses the depots.

Generally the number of depots built and the amount of flow using the depots decreases as the plant to demand cost $t_{ik}$ is reduced while other factors remain constant. It can be seen that when the per unit per mile transport costs from depot to demand and plant to demand is equal there are still one third of the depots built and

25% of the flow goes via the depots. In this situation we are comparing the cost per unit $g_{ik}$ with $a_{ij} + u_j + c_{jk}$. With less aggregation of demand and a greater number of possible depot locations one would expect greater usage of the depot system since $a_{ij}$ will become more dominant.

The effect of commodities sharing the same distribution system can be investigated by comparing the combined system with 2 separate distribution systems. Table 2 shows the total cost Z, the number of plants j, for one set of values of the parameters.

TABLE 2

| f | u | $t_{ij}$ | $t_{jk}$ | $t_{ik}$ | Z | j | |
|---|---|---|---|---|---|---|---|
| 150 | .03 | .001 | .005 | .01 | 70700 | 9 | combined |
| 150 | .03 | .001 | .005 | .01 | 62230 | 9 | Commodity 1 |
| 150 | .03 | .001 | .005 | .01 | 9500 | 2 | Commodity 2 |

Some of the cost differential (62230 + 9500 - 70700 = 1030) is due to more depots being built in the separate system.

The work is being extended to 25 depot sites and 143 demand points. Each of the above runs took about 130 seconds on a CDC 6400 using the APEX 3 software.

## REFERENCES

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