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GENERALISED FUNCTIONAL FORM AND RANDOM COEFFICIENT REGRESSION IN TRANSPORTATION RESEARCH

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ABSTRACT: *There are two developments in statistical methodology which have been examined extensively in the econometrics and statistics literature but have not been applied to a great extent in transportation research. These are the use of generalised functional form in regression problems, and random coefficient techniques.*

The purpose of this paper is to present a brief survey of these areas, to examine the uses of these techniques on transportation related topics and to suggest further applications of the procedures to transport. The exposition of these areas includes more intuitive rather than technical derivation so as to make the paper as accessible as possible.

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INTRODUCTION

The past decade has seen a rapid increase in the development of new statistical techniques, many of which will eventually become everyday tools to the applied researcher. However, it is very difficult to keep up with all new developments so that the non-statistician must rely on the occasional survey article to keep abreast of possibly useful techniques. The sad part of this whole story is that many of the survey articles which appear assume much more technical sophistication than the applied researcher may have (or wants to use). With this in mind, the present article is designed to survey two areas of recent development in methodology which are interesting in their own right and are potentially useful in transportation research. Much of what is discussed here will be shorn of technical details in an attempt to provide an intuitive feel for what is going on.

The two areas for discussion are: (1) the use of the generalised functional form originally developed by Box and Cox (1964) and Tukey (1957), and (2) random or stochastic parameter regression methods. Although each of these techniques has followed its own separate development, it is of interest to note that recently, researchers have begun to combine them into more sophisticated methods (e.g. Murthy (1976)) but more of this later. The next two sections contain discussion of the abovementioned areas in turn, followed by an outline of the current work being done on extensions of the techniques and some general comments.

FUNCTIONAL FORM

As will be the case with our other topic, we will begin by assuming that the transportation researcher is interested in estimating a relationship between some dependent variable, Y, and a single independent variable, X. This is assumed only for simplicity as the methods discussed apply equally well to the case of more than one independent variable. We also ignore the possibility of there being a simultaneous system of equations since most empirical work, at least initially, begins with single equation estimation.

More formally, we can write the relationship of interest as

$$Y = f(X) \tag{1}$$

It has been pointed out by a number of researchers (e.g. Dhrymes, et.al. (1972)) that theory often does not suggest the appropriate functional form to specify in empirical work. The usual specifications chosen are the linear or linear in logarithms form given respectively by

$$Y = \beta_o + \beta_1 X + \mu \tag{2}$$

or

$$\ln Y = \gamma_o + \gamma_1 \ln X + \nu \tag{3}$$

where ln means natural logarithm and μ, ν are error terms. The log-linear form (3) is popular among some applied researchers because the estimate of γ_1 can be directly interpreted as the elasticity of Y with respect to X.

Since the correct functional form is rarely known, the choice is, in most situations, essentially arbitrary. However, to some researchers, it seemed more reasonable to allow the data itself to aid in the choice of functional form instead of making an *a priori* choice, and hopefully accumulate evidence from a range of studies to reinforce or dispel a particular empirical form. Tukey (1957), Box and Tidwell (1962) and Box and Cox (1964) were among the first to systematically analyse the problem of functional form in a regression context. A good summary of their basic contributions may be found in Zarembka (1974) which should be the starting point for the interested reader who wishes a good technical overview of the problem of functional form in empirical analysis.

The basic Box-Cox transformation which results in a generalised functional form can be defined by considering a positive variable X which has been transformed such that

$$X^{(\lambda)} = \begin{cases} (X^{\lambda}-1)/\lambda & \text{if } \lambda \neq 0 \\ \ln X & \text{if } \lambda = 0 \end{cases} \tag{4}$$

That is, as long as the parameter λ is non-zero, we define the transformed variable $X^{(\lambda)}$ as $(X^{\lambda}-1)/\lambda$. But since it can be shown that as λ approaches zero, $(X^{\lambda}-1)/\lambda$ approaches the natural logarithm of X, we define $X^{(\lambda)}$ as ln X at $\lambda=0$ so that our transformation is continuous for all possible values of λ.

Now, using the above definition of a transformation, define a stochastic relationship between Y and X as

$$Y^{(\lambda_1)} = \alpha_0 + \alpha_1 X^{(\lambda_2)} + e \qquad (5)$$

It is not particularly difficult to see that if $\lambda_1=\lambda_2= 1$, equation (5) reduces to the simple linear function (2) while if $\lambda_1=\lambda_2=0$, the log-linear form (3) results. Therefore, it appears that the usual linear and log-linear forms are simply special cases of the generalised form (5). The question is then, how does one estimate the values of λ_1 and λ_2 in equation (5)?

To answer this question, first consider the case where $\lambda_1=\lambda_2$. If we assume that the error term in (5) is normally distributed, then we can define a likelihood function[1] for a given sample (see, for example, Zarembka (1974, pp. 85-86). We then simply regress our transformed X for a range of different values of λ (e.g. from +2 to -2 by increments of 0.1) and choose as our optimal value of λ that one which results in the maximum value of the likelihood function.[2] We may then use the estimated regression coefficients to calculate elasticities using the formula

$$E_{YX} = \hat{\alpha}_1 \ (Y/X)^{-\hat{\lambda}} \qquad (6)$$

where E_{YX} is the estimated elasticity of Y with respect to X and $\hat{\alpha}_1$, $\hat{\lambda}$ are estimated α_1 and the optimal value of λ.

In other situations, we might set λ_1 equal to 0 or 1 and allow λ_2 to vary, or set λ_2 equal to some value and allow λ_1 to vary. Estimation then proceeds as above. If we allow λ_1 and λ_2 to vary separately (or, in a

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1 The name likelihood function is given to the joint probability function of the given sample. Maximising a likelihood function is based on the simple idea that a given sample could be generated by different populations and that a particular sample is more likely to come from one population than another. Therefore, maximum likelihood estimates are the set of population parameters which would generate the observed sample most often.

2 Spitzer (1978) has suggested that a modified Newton maximisation procedure may be preferable. See his article for more details as the issue is much too complex to discuss here.

multiple regression situation, allow all λ's to vary separately) the estimation procedure is much more complex since the number of possible combinations of λ's increases tremendously. Box and Tidwell (1962) have suggested some search procedures for this case which are discussed in Zarembka (1974). An econometric computer program developed by White (1978) is available to estimate this most general version of the Box-Cox generalised functional form, but the user is warned that the routine should be used with the utmost caution since it is quite costly in terms of computer time since many iterations are usually needed for convergence.

The basic Box-Cox transform given by (4) was defined for a positive variable X. The question arises as to what can be done if the researcher's model contains one or more variables which take on negative values. A more general transformation, named the Box-Tukey transform (apparently named by Gaudry and Wills (1977)) is defined as

$$(X+\mu)^{\lambda} = \begin{cases} ((X+\mu)^{\lambda} - 1)/\lambda & \text{if } \lambda \neq 0 \\ \ln (X+\mu) & \text{if } \lambda = 0 \end{cases} \qquad (7)$$

where $X+\mu>0$. In other words, the new parameter μ is simply a location parameter (as opposed to the power parameter λ) chosen to assure that $X+\mu$ is greater than zero for all observations. Gaudry and Wills (1977) have analysed this transform in the context of travel demand models but most empirical work has concentrated on the Box-Cox transform which is simply a special case of (7) with μ set equal to zero.

The present author is aware of only two applications of these generalised functional forms in transportation related research. Hensher and Johnson (1978) in a study designed to examine the appropriate external structure of variables entering choice models of travel demand (i.e. whether the travel related variables should optimally be entered as absolutes, differences, ratios, or ratios of differences to averages) used the elementary Box-Cox transform to estimate linear probability models[3] under the various variable structures. To illustrate their results,

3 A model where Y in equation (1) takes on the value 0 or 1. See, for example, Pindyck and Rubinfeld (1976), pp.239-243 for more detail.

consider their examination of the ratio configuration. Their data consisted of 320 observations on travel times and costs for train and car mode for the journey to work by residents in a northern suburb of Sydney in 1971. Results for the ratio configuration when the simple linear model (2) was hypothesized are given by

$$G(X) = \underset{(24.7)}{.7127} \quad \underset{(-0.789)}{-0.0262}\,(t_1/t_2) \quad \underset{(-11.95)}{-0.246}\,(c_1/c_2) \tag{8}$$

$$\bar{R}^2 = .303$$

where subscript 1 = car, 2 = train, t-values are in parentheses, and G(X) is the probability of choosing car. Of the four configurations examined, this was the worst result for the linear model (i.e. λ=1) in the sense that the $\bar{R}^2$ was lowest and the time ratio was insignificant as shown by the low t-ratio.

When the Box-Cox transformation is used, however, a very different result emerges. Their optimal estimate of λ is 0.05 which is significantly different from 1 (the linear form) using the asymptotic chi-square test suggested by Zarembka (1974). Denoting the optimal estimate of λ by $\hat{\lambda}$, their result is given by

$$G(X) = \underset{(14.39)}{.6186} \quad \underset{(-3.02)}{-0.1658}\,(t_1/t_2)^{(\hat{\lambda})} \quad \underset{(-10.53)}{-0.4011}\,(c_1/c_2)^{(\hat{\lambda})} \tag{9}$$

$$\bar{R}^2 = .364$$

Not only does $\bar{R}^2$ increase, but the coefficient of the time ratio is now significant. Since the optimal λ of 0.05 is so close to zero, Hensher and Johnson conclude that, for their data set, the optimal functional form for the ratio configuration is logarithmic (i.e. λ=0), which is the same as differences of logs of the variables.

Gaudry and Wills (1977) were specifically interested in the functional form of travel demand models, but their excellent paper is much too complex to summarize briefly here. However, we can note that they estimate a cross-sectional intercity travel flow (or market share) model using a sample of 92 city pairs and an urban transit model estimated from time-series data. They extensively examined the effect of functional form on parameter estimates for both of their models and summarized their findings by stating, "We are brought to conclude that incorrect functional form specification may lead not only to incorrect elasticities but even to erroneous signs of important parameters." (Gaudry and Wills, 1977, p.2)

Therefore, further examination of the appropriate functional form in transport studies seems quite warrented. The actual implementation of the Box-Cox method in empirical work should not be particularly difficult since there are computer programs available which estimate models using the Box-Cox transform, most notably the program by Chang (1977) which has been specifically designed for Box-Cox estimation, and the econometric package SHAZAM (White 1978) which has both the Box-Cox and more general Box-Tidwell transform (i.e. all λ's allowed to vary separately) as options.

RANDOM COEFFICIENTS

Leaving the issue of functional form aside, let us assume that we are interested in a linear relationship between Y and X given by

$$Y_i = \beta_0 + \beta_1 X_i + \mu_i \quad (i=1,\ldots,N) \qquad (10)$$

which is essentially the same as equation (2) but where we now explicitly recognize that there are N observations on both Y and X. An assumption that is usually made in order to simplify estimation is that the coefficients (i.e. β_0 and β_1) in (10) are fixed across all N observational units. In fact, this assumption is often made without really thinking about its consequences (Johnson, 1977a). About 30 years ago, Wald (1947) suggested that in some cases it might be more appropriate to assume that regression coefficients are not fixed but are random variables. This seems reasonable in a cross-section context, for example, since it is hard to believe that each individual units response to a change in some stimulus (X) will be identical (i.e. β_1 fixed over N). Not much work was done in the area of random coefficient regression from the time of Wald's comment until the late sixties, but since that time, the literature on random (or stochastic) parameter regression has virtually exploded with now well over 100 references (see Johnson, 1977b, 1978).

One of the earliest random coefficient models was developed by Hildreth and Houck (1968) and is generally known as the Hildreth-Houck (H-H) model. It basically assumes that in an equation like (10), β_1 is not fixed but is replaced by β_{1i} where $\beta_{1i} = \bar{\beta}_1 + \nu_i$. In other words, the parameter or coefficient is a random variable with mean $\bar{\beta}_1$. The ν_i are random components with mean zero and variance σ_1^2. Extension to the multiple regression case is straight forward and is not discussed here. The estimation problem in the H-H model is to estimate what have become known as the mean response coefficients ($\bar{\beta}$'s) and the variances of the random β's (e.g. σ_1^2 above).[4] The most difficult part of H-H estimation is the estimation of these variances. Several alternatives have been suggested, many of which are discussed in Johnson (1977a, 1978), Raj (1975) and Froehlich (1973). It is not particularly instructive to examine these different variance estimators here, but whichever estimator is chosen, the resulting estimates are normally used in a generalised least squares estimator of the mean response coefficients (e.g. Johnson, 1978, p.76).

There is usually little quantitative difference between H-H estimates of the mean response coefficients and ordinary least squares (OLS) fixed coefficient estimates. The usefullness of the technique stems from the information it provides as to which coefficients in an equation are relatively more variable. For example, Johnson and Hensher (1978) have looked at the determinants of shopping trip frequency in a cross-section context and found that the occupation of the shopper was a more variable determinant of trip frequency than other explanatory variables that were important. They did this by examining the estimates of the variances of the random β's relative to the size of the mean response coefficient estimates (i.e. the coefficient of variation). We should note here that they also found a number of the variables in their study had coefficients which were fixed. The H-H model provides this information since a negative estimate of a variance (which is possible in the H-H model) implies that the corresponding coefficient is probably really fixed with a variance of zero (see Johnson (1978) for details).

The H-H model can be used for both time series and cross-section data but may be more appropriate in

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4 The variances of the random β's should be distinguished from the estimated variances (or standard errors) of the mean response coefficients, $\bar{\beta}$'s. This distinction is sometimes confusing (Burns, 1976).

the cross-section case, particularly since other stochastic parameter models have been developed exclusively for time series. One of these, the so-called Kalman filter, was originally developed in the engineering literature. The model seems potentially quite useful in many situations but is a bit too complicated to discuss here. The basic idea, however, is that the coefficients are updated from one time period to the next by incorporating the additional information provided by the next data point. The interested reader may find the recent article by Otter (1978) an excellent starting point since Otter takes pains to relate the Kalman filter model to ordinary regression.

The time-varying parameter technique that seems to be most popular is the Cooley-Prescott model (Cooley and Prescott, 1973, 1976). In this model, a parameter, say β_1, is assumed to vary in any time period around some permanent mean component and the permanent component to vary over time. Formally, the parameter variation scheme is defined by

$$\beta_{1_t} = \beta^*_{1_t} + \mu_t$$

$$\beta^*_{1_t} = \beta^*_{1_{t-1}} + \nu_t \qquad (11)$$

where β^*_t refers to the permanent component and μ_t, ν_t are the stochastic error terms. For the entire set of coefficients in an equation, the covariance structure of the set of coefficients which are all assumed to vary in the same manner as (11) is given by

$$\mathrm{cov}(\mu_t) = (1-\Theta)\ \sigma^2\ \Sigma_\mu$$

$$\mathrm{cov}(\nu_t) = \Theta\ \sigma^2\ \Sigma_\nu \qquad (12)$$

where $0<\Theta<1$ and Σ_μ, Σ_ν are covariance matrices which are assumed <u>known</u>.[5] The estimation problem is to find appropriate values for Θ, σ^2 and the values of the permanent components (β^*'s) for some chosen time period. Cooley and Prescott suggest choosing the first post-sample period since it is this period which might be of

5 Cooley and Prescott have found that their estimator is rather insensitive to misspecifications of Σ_μ and Σ_ν. Therefore, diagonal matrices are usually chosen in practice.

interest in forecasting, although any within the sample period will do. The details of how this is done will not be discussed here (see Cooley and Prescott, 1973,1976, or Maddala, 1977, Ch.17).

The estimated value of Θ is of particular interest in the Cooley-Prescott model since a value close to zero, implies that the parameters of the model are stable over time. This conclusion is reached since $cov(\nu_t)$ will be close to zero (see equation (12)).

An empirical study related to transport using the Cooley-Prescott model has been carried out by Schou and Johnson (1978) who estimated a demand for petrol function for the period 1955-1976 in Australia. They concluded that the demand for petrol function was reasonably stable over that period and that the short-run elasticity of demand for petrol in Australia was at most -0.08, suggesting that an increased petrol tax is possibly of very little help as a conservation measure. Because of the intuitive appeal of the C-P model and its relative computational simplicity, we may see a large number of empirical studies using Cooley-Prescott appearing in the transport related literature. To aid in this, a computer program developed by Bouwman and Prescott (1974) is available and is quite easy to use.

We have been looking at situations where the researcher is faced with a single cross section or time series of data. If there is a pooled data set (a time series of cross sections), then other random coefficient models may be more appropriate. Rosenburg (1973) has developed a model which allows both cross and time varying parameters but assumes that parameters converge towards some population mean over time. Hsiao (1974, 1975) describes a model with time and cross varying coefficients but his model seems rather difficult computationally. However, Swamy's work is probably the most well known in this context. His models allow for cross varying parameters (Swamy 1970, 1971, 1973, 1974) or for time and cross varying coefficients which are computationally simpler than Hsiao's (Swamy and Mehta 1975, 1977). Since a concise discussion of Swamy's work may be found in Johnson (1978) we will not discuss his models here. A simple computer program has been developed which estimates some of Swamy's less complex cross varying parameter models (Johnson and Oakenfull 1978).

Mehta, Narasimham and Swamy (1978) have examined a dynamic demand function for gasoline (petrol) for the United States using a pooled data set covering the period 1963-1973. Among other estimators employed, they used one of Swamy's random coefficient estimates which they concluded was the most appropriate for their study. Although the procedure used and model estimated was more complex than the Cooley-Prescott model estimated for Australia by Schou and Johnson (1978), their estimated price elasticity of -0.044 agrees reasonably well (i.e. is very small) with the Schou-Johnson estimate of -0.08 for a different country.

There is a lot more to random coefficient methods than implied by the above short discussion. Econometricians are applying (or attempting to apply) these techniques to a large number of situations. As a case in point, the reader interested in simultaneous systems of equations might look at the papers by Kelejian (1974) and Raj, Srivastava and Ullah (1978) on the use of random coefficients in that case.

Furthermore, there has been a sharp increase in the use of individual choice modelling techniques, particularly in transportation research, during the last decade (e.g. see Hensher 1978). The conditional logit model has been used extensively in this context, basically because of intuitive appeal and ease of estimation. Recently, Hausman and Wise (1978) have developed a conditional probit model for multiple choice situations which overcomes some of the severe restrictions imposed by logit (e.g. independence from irrelevant alternatives - see Hensher 1978). Although their model is more difficult to estimate, they show that it is feasible in many situations.

The Hausman-Wise model is essentially a random coefficient probit model. Fischer and Nagin (1978) conducted an experiment to compare the random coefficient probit model with a fixed coefficient model and concluded that the additional complicating assumption of random coefficients was probably well worth the extra computational burden. We will again refrain from any specific discussion of the models being discussed, referring the reader to the relevant sources if interested. Recall that the intent of this paper is to keep from getting too technical.

EXTENSIONS AND SUMMARY

Finally, we turn our attention to the recent work being done in combining the methodologies discussed above. Murthy (1976) has suggested an estimator for the Hildreth-Houck random coefficient model using the Box-Cox transform. Murthy's work stops short of any actual empirical work, simply providing an outline of an estimation procedure. It would certainly be useful to see some empirical results as well as some experimental evidence on the finite sample properties of the estimator. The present author knows of at least one project which investigated these matters (Hogan 1978). There are, as yet, no studies that have appeared which investigate the use of the Box-Cox or Box-Tukey transforms in other random coefficient models such as the Cooley-Prescott or Swamy models. However, unless more detailed results for the Murthy estimator are encouraging, it may be too much to expect the Box-Cox transform to work in more complex random parameter models.

The use of generalised functional form and random coefficients in qualitative choice models has progressed a bit further. Hensher and Johnson (1978) have examined the use of Box-Cox transformations in the linear probability model with reasonable results. Gaudry and Wills (1977, 1978) have tested the use of the general Box-Tukey transform in conditional logit models and in the modification of the logit model (named dogit) developed by Gaudry and Dagenais (1977). Gaudry (1978) has even explored the properties of the inverse Box-Cox and Box-Tukey transforms[6] in logit and dogit models. The present author is engaged in an analysis of the Gaudry-Wills models with particular emphasis on their implementation with disaggregate survey data since they used aggregate cross section data to test the logit and dogit specifications.

As far as random coefficient qualitative choice models are concerned, the recent work by Fischer and Nagin (1978) seems encouraging. They have empirically compared fixed with random coefficient probit models and have concluded that the added complexity (and subsequent cost) may be justified since the random coefficient model allows for a wider degree of variation in tastes across the sample. Therefore, extensions of their work may prove fruitful, as well as the possibility of applying random coefficients to the logit or dogit models, an area this author is also persuing.

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6 We will not get into this subject here. See Gaudry (1978).

7 This research is being sponsored by the Australian Road Research Board.

The next step is to ask whether a qualitative choice model with random coefficients, defined using a Box-Tukey transform might be superior to any of the models already discussed. One can imagine that eventually, somebody will investigate this possibility. However, there must be a limit to what we can ask our models to do for us. Maddala (1977, p 403) in the context of varying-parameter models sums up this thought quite nicely:

> "... the more general the models, the 'wollier' the questions we ask, and if we ask 'woolly' questions all we can expect to get are 'woolly' answers."

In this paper, we have endeavoured to provide a brief guide to two areas of recent (and rapid) development in statistical methodology. This was done, as far as possible, at a non-technical level so that the reader can get a feel for what is going on and be better equipped to tackle the more technical literature if the need arises.

The two areas of interest cover the use of (i) generalised functional form and (ii) random coefficients. Each of these topics was developed as simply as possible, some empirical results from transport referred to (although not discussed in much detail) and a guide to further reading provided. We also suggested sources of computer programs for many of the techniques discussed. Finally, we presented a short section on the combination of the methods and pointed out areas of possible future research.

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