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NETWORK CAPACITY FOR A SPECIFIC DEMAND

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ABSTRACT: *A computer program has been developed to use the Ford/Fulkerson algorithm to identify flow augmenting paths through a network and the minimum cut set of links for specific origin/destination pairs. A study has been made of an urban road network and the effects of change in network parameters, estimated in terms of changes in the volume/capacity ratio for the origin/destination demand matrix. A reduced network has been analysed to estimate the maximum circulation capacity of the network.*

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NETWORK CAPACITY

INTRODUCTION

In considering single elements of a network, such as a length of road or an approach to an intersection, an accepted measure of the level of service provided by that element, is the ratio of the demand volume to the capacity volume of the element. The Highway Capacity Manual (H.R.B. 1965) develops this concept in detail for both a length of road and for an approach to a signal controlled intersection. The concept is well based in systems analysis, deriving from the exponential relationship that has been shown to exist, both theoretically and empirically, between delay in a system and utilization of a system.

Intuitively, one could expect, that for the individual moving through a network on a trip between a specific origin, and destination, a measure of the level of service provided by the network for that specific trip could also be characterized in similar fashion, by the volume demand for that specific trip, to the capacity of the network to satisfy the demand for that trip. This concept is further developed and is applied to a study of the network serving a shopping centre in the Sydney suburb of Caringbah.

DESCRIPTION OF A NETWORK

As a first step in defining the capacity of a network, the network needs to be described in formal and rigorous terms. A common approach described in the literature, is to describe the network in terms of links and node points. A simple analogy is to a length of road as a link and an intersection as a node point. For all but the simplest networks however, some further refinement of definition is needed.

A link may be considered as either directed or undirected. If a network of roads is to be modelled, there are advantages in using directed links, which means that any two way section of a road must be modelled as two separate directed links, giving rise to the definition that a link models a particular movement between nodes. The capacity of the link is conceptually straight forward and can be stated as the maximum volume flow that can traverse the link during a specified interval of time.

A further characteristic of the network that must be determined is the topology or connectivity of the network which is a statement of how the nodes are interconnected. Such information is usefully represented by a node-link incidence matrix (Blunden 1971).

In considering the node points of the network; conceptually, they may be visualized as points in space at which access from one link to another connected link is possible. The definition of node capacity is however a more difficult task. In the established theory of network flows, it is common to assume that only the links are capacitated and that the capacity of a node point is infinite. If this concept is accepted, then in applying the theory to practical cases, the problem arises that in road networks in particular, the intersections offer the critical constraints to flow through the network, so that the analogy between intersection and node point is not a valid one.

Potts and Oliver (1972) have discussed this problem and suggest that an intersection can be modelled as a set of dummy node points, each dummy node representing an approach to an intersection, connected by a set of dummy links, where each link represents a specific movement through the intersection and to which a capacity can be given.

Capacity of a Link

Signalized Intersection. Miller (1968) has given a methodology to determine the capacity of an approach to a signalized intersection which has been adopted in this study. A saturation flow rate Si has been determined from the geometry of the intersection for each movement using the standard values and correction factors given by Miller.

Priority Controlled Intersection. The majority of intersections in the network, are priority controlled, either through provision of giveway or stop signs or simply through application of the "give way to the right" rule. For such intersections, the capacity of the major traffic stream is governed by the geometry of the approach and has been taken as the saturation flow rate of the equivalent signalized approach.

The capacity for movement from the minor stream approach however, is governed by the flow occurring in the conflicting major traffic stream. For a given flow q veh/hour in the major traffic stream, an estimate of the absorbtion rate of conflicting movements is given by the well known formula

$$p = q(1-EXP(-qT))/(1-EXP(-qt))$$

where p is the absorbtion rate and which in this study is equated to the capacity of the link modelling the minor stream movement

- q is the major stream flow rate
- T is the critical gap for the particular conflicting movement
- t is the follow up headway from the minor stream approach.

For the purposes of this study, as a simplification, T has been taken as 4.5 seconds and t as 2 seconds for all possible movements. Where the minor stream interferes with two major stream flows, as in a right turn or straight through movement, then q has been taken as the sum of the two conflicting flows.

Capacity of the Network

The capacity of a network can be defined in meaningful terms only with respect to a specific origin and destination. Ford and Fulkerson (1962) introduced the idea of a cut set in which the set of nodes in the network is divided by an imaginary cut, into two complementary sets; X which contains the origin node and $\bar{x}$ which contains the destination node. The capacity of such a cut is defined as the sum of the flows on those links which connect the nodes in the destination set $\bar{x}$ to the nodes in the origin set X. Ford and Fulkerson have shown, that the maximum flow that can occur between the specific O/D pair, is the minimum cut capacity of all possible cuts.

As an illustration of this concept, Figure 4 shows a very simple network with four nodes and five directed and capacitated links.

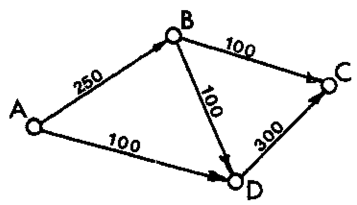

FIG 4

If node A is considered to be the origin and node C the destination, the possible cuts are:

Cut	X	$\bar{x}$	Cut Capacity
1	A	B,C,D	350
2	A,B	C,D	300
3	A,B,D	C	400

The network capacity between A and C is the minimum cut capacity and so is equal to 300.

The critical links or "cut set" which limit the flow, and so determine the capacity, are the links (A, D), (B, D) and (B, C). In this simple case, it is obvious that the link flows will be 100 on links (A, D), (B, C), (B, D) and 200 on links (A, B) and (D, C).

Initial Set of Flows

Depending on the way in which the demand for travel between all feasible O/D pairs has assigned itself to the network, there will be a volume flow along each link in the network. These volume flows for the particular demand and assignment are taken as the starting point for the subsequent network capacity analysis.

Flow Augmenting Paths

For a given set of flows on links in the network, if the capacity for each and every link along a path connecting a specific O/D pair exceeds the current link flow, then there must be a capacity for additional flow along that path. Such a path is termed a "flow augmenting path". Conversely, when no such flow augmenting path can be found, then the link flows of the minimum cut set define the capacity for movement between the O/D pair.

Ford and Fulkerson (1962) have given a labelling algorithm which searches for such flow augmenting paths and which also determines the minimum flow cut set. It should be noted, that if an initial set of link flows is assumed, then the labelling algorithm searches for flow augmenting paths to the existing flows. The capacity is then the existing flow plus the augmenting flow.

Using the algorithm, it is possible to look at how an increase in demand between an O/D pair could be loaded on to the network and also to assess the effects on capacity between each O/D pair of changes in network parameters. If the vol/cap ratio is taken as a measure of the level of service provided to a specific O/D demand, then the comparative effects of change to the network can be assessed.

Reduced Networks

The capacity for flow between specific O/D pairs is highly dependent on the demand that exists between all other O/D pairs, since paths between such O/D pairs must share common links. The coupled nature of the network capacity and the O/D demand matrix can be studied by defining a reduced network. (Simonard 1966).

If a single hypothetical source is assumed connected to all possible origins by directed links of infinite capacity and a single hypothetical sink is assumed connected to all possible destinations again by directed links of infinite capacity, then the capacity flow between the source and the sink, represents the maximum flow that can occur through the network assuming that the distribution of demand is unspecified. With a suitable definition of origin and destination nodes, such a capacity between source and sink can be thought of as the capacity for circulation in the network.

COMPUTER PROGRAM

A computer program has been written in SIMULA which uses the Ford/Fulkerson Algorithm to identify flow augmenting paths through the network. The input data to the program is a complete description of the network in the form of nodes and links. Each link modelling a major movement, is assigned a fixed capacity and each link modelling a minor movement is assigned the identities of the conflicting major streams. The definition of capacity for the minor movement is then recursive as each augmenting flow is added to the current link flow. The topology of the network is completely defined by associating with each node, a set of connected nodes.

SIMULA is a derivative of ALGOL 60 and is a language which is particularly efficient in dealing with sets, the theory of which underlies network theory in general.

CASE STUDY

Figure 1 shows the geography of the road network that serves the shopping centre in the Sydney suburb of Caringbah. During the morning peak there is a marked through traffic demand which moves primarily from south and east to north and west proceeding either to Miranda or Taren Point. The through traffic is mixed in with local traffic circulating in the area.

Caringbah Network

Figure 2 shows the node link representation of a section of the street network. There are two sets of traffic lights controlling a complex intersection, the detailed representation of which, in the form of dummy nodes and links is shown in Figure 3. The two sets of lights are traffic actuated with upper bounds on the green time allocated to each phase. For saturated conditions, the lights operate in the fashion of fixed time signals and since it is capacity that is being estimated, the upper bounds have been used to determine the maximum effective green time allocated to each phase and hence to each particular movement. As stated before, the capacity of a dummy link representing a particular movement has been calculated as $S_i\, g_i/C_t$. There are 113 nodes and 185 links used to represent all movements through the network.

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TABLE 1

NETWORK CAPACITIES

Network 1

ORIG.	DEST.	DEMAND	Xcap	CAP.	VOL/CAP.	CUT SET
57	162	530	179	709	0.75	(721,724)(723,724)
31	162	520	179	699	0.74	(721,724)(723,724)
83	162	80	179	259	0.31	(721,724)(723,724)
57	82	430	534	964	0.45	(711,7111)(717,7171)(717,7172)
1000	2000	5800	3190	6990	0.64	(713,7131)(714,722)(721,724) (723,715)(723,724)

Network 2

ORIG.	DEST.	DEMAND	Xcap	CAP.	VOL/CAP.	CUT SET
57	162	530	295	825	0.64	(721,724)(723,724)
31	162	520	295	815	0.64	(721,724)(723,724)
83	162	80	295	375	0.21	(721,724)(723,724)
57	82	430	534	964	0.45	(711,7111)(717,7171)(717,7172)
1000	2000	5800	3285	7085	0.64	(717,7172)(81,82)(165,166) (165,162)

Network 3

ORIG.	DEST.	DEMAND	Xcap	CAP.	VOL/CAP.	CUT SET
57	162	530	110	640	0.83	(721,724)
31	162	520	110	630	0.82	(721,724)
83	162	80	110	190	0.42	(721,724)
57	82	430	534	964	0.44	(711,7111)(717,7171)(717,7172)
1000	2000	5800	3120	6920	0.65	(713,7131)(7141,722)(721,724) (723,715)(717,7172)(81,82) (165,166)(165,162)

Network 4

ORIG.	DEST.	DEMAND	Xcap	CAP.	VOL/CAP.	CUT SET
57	162	530	250	780	0.68	(721,724)
31	162	520	250	770	0.68	(721,724)
83	162	80	250	330	0.24	(721,724)
57	82	430	534	964	0.45	(711,7111)(717,7171)(717,7172)

Note: Xcap = Capacity - Flow
1000 identifies the source of the reduced network
2000 identifies the sink of the reduced network

TABLE 2

CHANGES TO LINK CAPACITIES

LINK	NET 1	NET 2	NET 3	NET 4
(713,714)	373	410	373	410
(711,721)	1200	1080	1200	1080
(721,724)	1140	1280	1140	1280
(723,724)	240	216	Deleted	Deleted