

ESTIMATION OF AN AGGREGATE PRODUCTION FUNCTION USING POOLED CROSS-SECTION
TIME-SERIES DATA FOR AUSTRALIAN RAILWAYS

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ABSTRACT: *The aim of the study is to estimate a production function representing the technological relationship between output and factor inputs.*

The virtue of estimating a production function is that it provides a better indication of capital and labour productivity, because it shows the separately attributable increments of output due to a unit increase in labour and to a unit increase in capital. It also provides a measure of the true marginal factor productivity, which is vastly superior to input-output ratios which fail completely to distinguish between the contributions of the factors to output.

RAILWAY PRODUCTION FUNCTIONS

The study attempts to further the insights of production relations given by input-output ratios through the estimation of an aggregate production function.

The production function provides a better indication of capital and labour productivity because it shows the separately attributable increments of output due to a unit increase in capital and to a unit increase in labour. But if output is only expressed as a simple ratio to labour and capital respectively, the separately attributable contributions of these two factors to output are not identified. The fixed proportions of input-output ratios suggest that the underlying form of the production function is a right angle or Leontief function, indicating that there is no possibility of substitution between factor inputs. This assumes that the conditions of production are such that once the level of output is given, the quantity of the inputs are uniquely determined. However, production theory argues that the amount of each input used in providing a given output will respond to changes in the relative prices of the inputs. It is this neglect of factor substitution and the inability to provide the true marginal factor productivities which distinguishes between the contributions of the factors to output making the estimation of a production function a vastly superior method.

PREVIOUS RAIL STUDIES

Railway productivity relations have been analysed using both production and cost function estimation techniques. In practice, measuring cost curves is often more convenient than estimating the production function since the available accounting data are normally reported in money terms. Both functions have been estimated using a variety of functional forms from the restrictive Cobb-Douglas, used by Borts (1960), Friedlaender (1971) and Kneafsey (1975) to the more flexible translog function used by Oum (1979) and Caves, Christensen and Swanson (1980) to name a few.

Kneafsey (1975) estimated production elasticities to determine whether or not economies of scale existed in the railway industry. His analysis of five railroads, using time-series data suggested that there were increasing returns to four of the five railroads. His input measures were identified as the number of crews and investment in plant and equipment with output being measured in gross-ton-miles.

Borts (1960) estimated long run cost elasticities in order to determine whether there were increasing returns to scale in the railway industry. Using American data, he was able to estimate two cross-section models, one for the line haul process, the other for the switching process. He separately identified outputs into loaded and unloaded freight car miles. Inputs included man-hours employed, fuel consumption, expenditure on maintenance of freight equipment and track, and miles of track, both less depreciation. Both he and Friedlaender (1971) took account of excess capacity in their models. Keeler (1973) went one step further by estimating a cost function using a cross-section sample of fifty railways to estimate the appropriate amount of excess capacity in the industry, and the amount of money that could be saved by individual railways, by abandoning it.

Caves, Christensen and Swanson (1980) estimated a cost function using time-series data for Class I railroads. They used the more flexible translog function which separately identified passenger and freight outputs using a weighting system to take into account variations in the

length of haul. Inputs were specified as labour man-hours weighted by seven occupational groups. The capital stock index was derived by using the perpetual inventory method and was divided into way and structures, and equipment. Other inputs specified were fuel and materials.

The disaggregate nature of these studies is attributable to the more efficient methods of collection and reporting of railway data by the Inter-state Commerce Commission.

DEVELOPMENT OF THE STUDY

Logically one should proceed by first using the Constant Elasticity of Substitution (CES) production function to test whether the elasticity of substitution of capital for labour in railways is significantly different from unity. If it is not, then it is appropriate to proceed with the estimation of the Cobb-Douglas function, which assumes an elasticity of substitution of unity.

Studies on the substitution between labour and capital using the CES for individual industries, based upon cross-section data from 19 countries (Fuchs, 1963) reported estimated elasticities of substitution ranging from 0.66 to 1.32, but only the result for the glass industry ($\sigma = 1.27$) is significantly different from unity.

Estimates for CES functions are extremely sensitive to data specification: the estimates of σ tend to unity or greater when cross-section data are used, and nearer to one half when time series data are used. These differences are attributed to the nature of the technology of the firm (Johnston 1960). Studies which have used cross-section data have obtained estimates indicating that substitutability of inputs in the longer run is far greater because the firm has had time to plan and make the necessary changes to factor inputs in response to changes in relative factor prices. By contrast, time series studies indicate that the substitution possibilities have diminished after new capital stock has been installed, exhibiting relatively low elasticities of substitution.

Another important functional form which allows the elasticity of substitution to vary, is the transcendental logarithmic production function, which allows the elasticity of substitution to vary with factor proportions. Work by Oum (1979) on road-rail substitution in Canada, which treated road and rail transport as inputs to a general production process, gave results which generally did not deviate from $\sigma = 1$ at the proportions actually prevailing.

However, it has been found from previous studies that the Cobb-Douglas provides an extremely good "average" model of an industry's production processes.

CROSS-SECTION VS TIME-SERIES ESTIMATION

In general three types of analytical approaches are possible, the extent to which each can be pursued being dependent upon data availability. Firstly, there is the estimation of a cross-section production function describing output for railway firms at one point in time. With the basic presupposition that each firm has had ample time to adjust its capacity in terms of capital equipment and plant

to its particular circumstances.

The advantage of an inter-industry cross-section study is that one does not have to deflate the value data, which is one of the major problems associated with time series analysis. This is not true, however, for international cross-section studies since the value data for different countries must be deflated to a common monetary unit.

The second approach is based upon the analysis of time-series data. One suspects that the specification is itself too simple, because one knows that adjustment of the capital stock takes time and that the technological efficiency of new capital is greater than that of old. Previous results suggest that the most important reason is the unsatisfactory nature of the data, in particular, the capital stock series with its associated index problems. There is also the added difficulty that the estimated co-efficients may be measuring technical progress and not the production function.

Lastly, one can pool cross-section and time-series data, which is specifically appropriate if the number of observations available for the sample are small. However, this method is not without its own problems, especially concerning the interpretation of the estimated production elasticities i.e., cross-section estimates are long-run while time-series estimates are short-run elasticities.

DATA SET

There are two major constraints that complicate the estimation of a production function for Australian Railways, (1) the unsatisfactory state of the data, which is attributable to the lack of standardisation of data reported by railway systems, and (2) the sample is limited to eight railway systems.

The published data sources amount to the following:
Rail Finances: Supplementary Paper No. 3 ARKDO 1981; Rail, Bus and Air Transport Australia, A.B.S. Cat. No. 9201.0, Railway Annual Reports for each system and New Zealand.

The lack of a standardised value for capital stock has caused the initial sample of eight to be divided up into three sub-groups according to the capital stock variable that could be identified (1) Data for Australian National Railways, New South Wales, New Zealand, South Australia, Tasmania and Western Australia. Observations are limited to these systems because Queensland and Victoria do not report values for Fixed Assets in their balance sheets, (2) Data for New South Wales, New Zealand, Queensland, South Australia, Victoria and Western Australia can all be grouped together using Queensland's comparative analysis, which reports Capital Open Lines for only these railway systems, (3) A perpetual inventory series was constructed from the physical measures of capital assets reported by New South Wales, Queensland, South Australia, Tasmania and Western Australia. A complete inventory series was not available for Victoria and New Zealand, and ANR do not report inventory measures.

All observations for the sample were taken prior to the amalgamation of Australian National Railways with South Australia and Tasmania to form the Commonwealth Railways in 1977. After this period the number of railway systems are effectively reduced to six.

Tonne-kilometres would have been a more satisfactory measure of output, because it would take account of increasing vehicle capacity and load factors where there are clear distinctions between the physical characteristics of the goods carried, and therefore in their loading and stowage characteristics. This measure could not be used in the analysis because (1) conversions of passenger kilometres to tonne-kilometres are arbitrary and (2) passenger kilometres are not reported for all railway systems in the sample.

While the inadequacies of the data are very serious, there seems to be fewer problems in the cross-section data than in the time-series analysis, as it is unnecessary to deflate money values.

MODAL SPECIFICATION

There are many functional forms that may serve as production functions. But none have the simplicity of the CES and Cobb-Douglas form. Nearly all research into empirical relationships have used one or other of these functions as a standard model.

The simplest functional form that will have a consistent elasticity of substitution that may take any admissible value, between zero and infinity is the CES which is specified thus:

$$Y = \gamma [\delta K^{-\rho} + (1-\delta) L^{-\rho}]^{-\frac{1}{\rho}} e \quad (1)$$

$$(\gamma > 0; 1 > \delta > 0; \rho > 0; \rho \geq -1)$$

where Y, K and L are output, capital and labour respectively and γ , δ and ρ are non negative parameters to be estimated using a non-linear maximum likelihood estimator. The elasticity of substitution can be calculated thus:

$$\sigma_{KL} = 1/(1+\rho)$$

Alternatively one can use a linear regression on a Taylor series approximation (Kmenta, 1971) to estimate a CES production function which is represented thus:

$$\ln RTK = \beta_0 + \beta_1 \ln L + \beta_2 \ln \left(\frac{K}{L}\right) + \beta_3 [\ln \left(\frac{K}{L}\right)]^2 + e \quad (2)$$

where

$\ln RTK$ = natural log of revenue train kilometres,

$\ln L$ = natural log of labour,

$\ln \left(\frac{K}{L}\right)$ = natural log of the capital labour ratio,

$[\ln \left(\frac{K}{L}\right)]^2$ = natural log of the capital labour ratio squared,

e = disturbance term,

β_0 to β_3 = parameters to be estimated,

One tests the condition $\beta_3 = 0$, in order to determine the power of the approximating function in being able to discriminate between the Cobb-Douglas and the CES form. There are biases inherent in the Taylor series approximation (Thursby & Lovell, 1978), however its results can be successfully used as starting values for the maximum likelihood method. The parameter values of equation 1 can be readily

determined from equation 2 thus:

$$\ln \hat{Y} = \beta_0$$

$$\hat{\sigma} = \beta_1$$

$$\hat{v} = \beta_2 / \beta_1$$

$$\hat{\rho} = \frac{-2\beta_1\beta_3}{2(\beta_1 - \beta_2)}$$

The traditional form of the production function which has been found to give satisfactory approximations to a wide variety of industries is the Cobb-Douglas of the form:

$$Y = AL^{b_1}K^{b_2} \quad (3)$$

$$(Y > 0, A > 0, L > 0, K > 0)$$

where Y, K and L are output, capital and labour respectively, and b_1 and b_2 are the production elasticities to be estimated.

Underlying the use of the Cobb-Douglas is the fact that rationally organised firms will be operating at a level where returns to each input, with others held constant, will be diminishing. In addition to providing estimates of the marginal productivities of the inputs, one is also able to provide estimates of returns to scale.

Given the available data and sample size, using this simple production function will produce results which are not statistically reliable i.e., there are a maximum of six railway systems and with two independent variables (capital and labour) one is left with only three degrees of freedom or two if New Zealand is omitted. The degrees of freedom problem can be overcome by pooling cross-section and time-series data. One uses a sequence of annual observations on the six railway systems, with slope and intercept shifting dummy variables for each year. In this way separate cross-section estimates for each year are obtained, with the degrees of freedom being summed over the whole regression (Ben-David & Tomek, 1965). The equation is represented thus:

$$\ln Y_{it} = \sum_{i=6} a_t D_{ijt} + \sum_{i=6} b_i S_{jit} (\ln X_{it}) + e_t$$

where $\ln Y_{it}$ = the log of output of the i^{th} railway system in the t^{th} year,

$\ln X_{it}$ = the log of the independent variable of the i^{th} railway system in the t^{th} year,

D_{ijt} = intercept shifting dummy variable (j)

with $D_{ijt} = \begin{cases} 1 & \text{when } i = j \\ 0 & \text{when } i \neq j \end{cases}$

S_{jit} = slope changing dummy variable(j)

with $S_{jit} = 1$ when $i = j$
 $S_{jit} = 0$ when $i \neq j$

e_t = disturbance term,

$ij = 1, 2, \dots, 6,$

$t = 1972, 1973, \dots, 1977$

a_t and b_i are parameters to be estimated

Thus five years of data from 1972-73 to 1976-77 and six systems give a total of thirty observations. One degree of freedom is lost for each of the five yearly intercept shifting dummy variables (which together replace the usual constant) and one for each of the ten coefficients (two per year) making a total of fifteen. This leaves fourteen degrees of freedom. Clearly this procedure does not get over the requirement that there must be fewer independent variables than the number of railway systems in the regression minus one. But given the nature of the data it appears to be an acceptable estimating procedure.

This approach has three advantages: (1) the principle advantage lies in pooling the data to increase the degrees of freedom, (2) permits the estimation of separate annual cross-sectional relationships in one step, and (3) there is no need to deflate money values. This follows from the fact that a separate relationship is estimated for each of the five years. It is also important that the within-year data for each system be stated on the same basis as the data for each other system, but changes can be allowed between years. What matters is that one must be consistent when specifying the relative input and output values across systems within a year.

More complex models can be devised, for the coefficients of the production function to vary from one entrepreneur to another. For the i th entrepreneur the coefficients of labour and capital would be respectively b_{i1} and b_{i2} which would not be the same for all entrepreneurs. This may be rationalised by suggesting that some entrepreneurs are better, on the average, at more capital intensive production than others, while some may be more adept at organising labour intensive methods, which is equivalent to saying that there is no unique production function i.e., there are as many production functions as there are entrepreneurs. However although such a concept may be desirable, it would be extraordinarily difficult to deal with in practice. The less complicated way of determining the entrepreneurship assumption, is to suppose that the coefficients are the same for each firm, with similar entrepreneurial effects appearing in the marginal productivity conditions i.e., the efficiency of an entrepreneur is reflected not merely in his production function, but is also reflected in the precision with which he achieves the best employment of the factors.

However, if there were as many firms overproducing as there were underproducing (by the same amounts on average) the regression of output on one of the inputs will show more or less constant returns to scale. If on the other hand most of the mistakes were in underproducing then we would have a relative concentration of points on the production function indicating increasing returns to scale.

An important difficulty with most practical studies is that all factor inputs cannot easily be measured, or can be quantified only very roughly. Consequently, some factors of production are omitted from the analysis. The main difficulties occur in trying to quantify capital stock, so as to correspond to the capital services provided. The advantage of cross-section over that of time-series data is that the variations in the amount of idle capacity are probably less. Shepherd (1953) and Klein (1948) overcome this problem of idle capacity by specifying capital stock in the form of total train hours operated, as a measure of input. The main problems of measuring capital arise because capital consists of various kinds of rolling stock, buildings and land at different stages of their life cycles. Combining these assets into monetary measures involves the usual index number problems i.e., fixed price weights used in combining inputs cause distortions in the movement of the resulting aggregate index of inputs.

Fixed assets (less depreciation) and Capital Open Lines, both suffer from the same problem, that different railway systems use different accounting methods when depreciating capital stock over their service lives. Ideally the depreciation function should be the theoretically perfect depreciation method where the allowance in each year reflects the assets value to receive future earnings. There are additional difficulties which occur when stock units are reported in historic costs, (1) they do not reflect their current replacement costs, (2) consideration is not given to technological advances, and (3) no consideration is given to the fact that railways may have made unwise decisions resulting in investments in the wrong types of rolling stock. Thus these problems contribute to the difficulty of specifying accounting figures as proxies for the flow of capital stocks in a railway production function.

The easiest series to measure is labour which for this analysis has been specified in three forms, (1) wages plus on-costs, where on-costs represent long service leave, payroll tax, sick leave etc (which rose faster than average weekly wages), (2) wages paid from working expenses and, (3) total man-hours worked, which is believed to be a better proxy for productive working time when compared to (1) and (2) above. The estimate of man-hours suffers from having to multiply the total number of railway employees by the "average number of hours worked in the Transport and Communications Industry". Labour specified as total man-hours was weighted to reflect the changing mix between salaried staff and wages staff according to remuneration in the base year.

Gross output measures that are available from published reports differ considerably in their accuracies as indicators of output-value which is directly related to the multiproduct nature of the industry. For this analysis, the only consistent measure available for the analysis is total revenue train-kilometres, which is the sum of passenger and freight-train kilometres. This measure does have the disadvantage of not allowing for increased vehicle capacity or load factors, and does not distinguish between passenger and freight across systems where train-kilometres vary from short passenger trains to long distance coal and ore trains of thousands of tonnes. As a measure of output, it is not sensitive to increasing rail weights, larger capacity freight wagons and the employment of increasingly more powerful locomotives. Trains are achieving much higher load factors so that net tonne kilometres are increasing while train kilometres are decreasing.

RESULTS

Estimation of the Taylor series approximation (equation 2) indicated that $\beta_3 = 0.029$ (t-statistic = 0.07) was not significantly different from zero, suggesting that the Cobb-Douglas would be a suitable functional form. The parameter values were calculated and used as starting values for a maximum likelihood estimation of equation 1. Estimation of the CES function was unsatisfactory, due to the presence of multicollinearity between capital and labour, which has caused the maximum likelihood surface to be very flat so that convergence to an optimal point was not possible.

Results for the Cobb-Douglas are presented in Table 1. Column (i) shows the estimated coefficients for the multiplicative constant term, column (ii) shows production elasticities for labour, specified as wages plus on-costs, wages paid from working expenses and total man-hours worked respectively and column (iii) shows the production elasticities for the capital stock, column (iv) shows returns to scale.

TABLE 1 ESTIMATED CROSS-SECTION RAILWAY PRODUCTION FUNCTIONS, BASED ON DATA FOR AUSTRALIAN NATIONAL RAILWAYS, NEW SOUTH WALES, SOUTH AUSTRALIA, WESTERN AUSTRALIA, TASMANIA AND NEW ZEALAND (CONVERTED TO EQUIVALENT AUSTRALIAN DOLLARS). (t-Statistics in brackets)

Year Ended 30th June	Elasticity of Production of Revenue Train-Kilometres with respect to			
	Constant (i)	Wages and On-Costs b_1 (ii)	Fixed Assets b_2 (iii)	Returns to Scale $b_1 + b_2$ (iv)
1973	0.122 (6.85)	0.749 (4.71)	0.317 (1.96)	1.07
1974	0.116 (6.88)	0.755 (3.96)	0.296 (1.54)	1.05
1975	0.078 (8.76)	0.687 (3.20)	0.387 (1.81)	1.07
1976	0.063 (9.94)	0.757 (4.27)	0.337 (1.88)	1.09
1977	0.069 (10.76)	0.826 (6.30)	0.255 (1.84)	1.08
R ² = 0.9951		df = 14		

RAILWAY PRODUCTION FUNCTIONS

TABLE 1 ESTIMATED CROSS-SECTION RAILWAY PRODUCTION FUNCTIONS, BASED ON DATA FOR AUSTRALIAN NATIONAL RAILWAYS, NEW SOUTH WALES, SOUTH AUSTRALIA, WESTERN AUSTRALIA, TASMANIA AND NEW ZEALAND (CONVERTED TO EQUIVALENT AUSTRALIAN DOLLARS). (t-Statistics in brackets) (cont.)

Year Ended 30th June	Elasticity of Production of Revenue Train-Kilometres with respect to:			
	Constant	Wages from Working Expenses b_1	Fixed Assets b_2	Returns to Scale
1973	0.127 (6.39)	0.740 (4.58)	0.328 (2.00)	1.06
1974	0.112 (6.84)	0.722 (3.92)	0.337 (1.83)	1.05
1975	0.074 (8.04)	0.720 (3.13)	0.365 (1.62)	1.08
1976	0.047 (12.47)	0.667 (3.96)	0.471 (2.94)	1.13
1977	0.063 (10.76)	0.852 (6.17)	0.251 (1.84)	1.11

$R^2 = 0.9941$

Year Ended 30th June	Elasticity of Production of Revenue Train-Kilometres with respect to:			
	Constant (i)	Man-hours Worked b_1 (ii)	Fixed Assets b_2 (iii)	Returns to Scale $b_1 + b_2$ (iv)
1973	0.183 (3.38)	0.693 (3.54)	0.408 (2.15)	1.10
1974	0.174 (3.25)	0.642 (3.03)	0.448 (2.21)	1.09
1975	0.125 (3.53)	0.544 (2.35)	0.552 (2.52)	1.09
1976	0.127 (3.74)	0.628 (3.16)	0.496 (2.57)	1.12
1977	0.161 (3.77)	0.763 (4.67)	0.395 (2.52)	1.13

$R^2 = 0.9966$ $df = 14$

Because of the possible inconsistency in using New Zealand Railways, a sensitivity test (omitting New Zealand from the sample) was carried out. The sensitivity test indicated that the New Zealand data did not unduly influence the estimated elasticities.

Results in Table 1 show the estimated production elasticities to be unstable between the separate annual relationships. This instability cannot be attributed to mis-specification caused by deflating data or from distortion from technical progress over time, but is more likely due to the inadequacy of the measures of output and inputs.

Judged by statistical criteria the estimated regressions give excellent fits ($R^2 > 0.9551$). The production elasticities are all statistically significant at the five per cent level, with the exception of the fixed assets variable for the years 1974, 1975 and 1976 in the first regression, and 1975 in the second regression which are only significant at the ten per cent level.

b_1 and b_2 measure the fraction of total receipts paid respectively to labour and capital. These elasticities can be used to give a very rough indication when attributing income shares, i.e., two-thirds of total income has been paid to labour in the form of real wages. Man-hours and wages give different results. If wages were simply man-hours multiplied by the one wage rate for a given year then the estimated production elasticities would be the same in both cases i.e., scaling an input variable does not affect the elasticity. The variation between the two sets of elasticities may be attributed to the measure of man-hours i.e., man-hours worked is believed to be a better measure of labour productivity, whereas man-hours paid reflects both productive time and non-productive time which is composed of holiday pay, sickness pay, overtime etc.

The addition of b_1 and b_2 indicates that railways exhibit increasing returns to scale ($b_1 + b_2 > 1$) i.e., increasing both inputs by 10 per cent will give rise to an 11 per cent increase in output. Roughly speaking, one may argue that increasing returns in railway systems indicates under-utilization of available capital stock. - Summing the production elasticities in this way to reach conclusions about returns to scale, one needs to be fairly sure that all inputs are summarised by the factors specified in the estimating model.

The estimates of the production elasticities are believed to be inefficient due to the presence of multicollinearity between the independent variables, and the existence of a heteroscedastic pattern of the error term. Multicollinearity is illustrated in Figure 1 where the capital-labour output surface has been plotted for each of the five railway systems for 1973 and 1977. One notes that observations are concentrated along a ray with very little variation in the capital-labour plane. This makes it very difficult to position the whole surface (except along the ray), i.e., one is only able to locate the slope in the Y direction. The t-statistics for the estimated coefficients indicate that given the presence of multicollinearity, the regression has enabled the individual influences of the independent variables to be measured with some reliability.

Capital Open Lines and the Perpetual Inventory Series both proved to be unsatisfactory, with coefficients exhibiting negative signs and so were excluded from further estimation.

RAILWAY PRODUCTION FUNCTIONS

The marginal productivities, which are the major outputs of this study are calculated for each system in each year, which enables one to attribute growth in output to the proximate causes. Estimates of the marginal products attempt to show the increase in revenue train-kilometres from a very small change in one of the factors of production. The assumption that the estimated production elasticities apply to each railway system is based upon the deviations between the actual and predicted estimates for X_i , where the estimated value (\hat{X}_i) represents the production function. The variances are so small that for all intents and purposes the estimated values are the same as the actual values indicating that all railway firms in the sample are on the same production function. The constant term allows the estimated production function to shift slightly so that it passes through all estimated values. On this basis, the marginal productivities are calculated for system i thus:

$$MP_L = b_1 \frac{\text{Revenue Train-Kilometres in System } i}{\text{Wages in System } i}$$

$$MP_K = b_2 \frac{\text{Revenue Train-Kilometres in System } i}{\text{Fixed Assets in System } i}$$

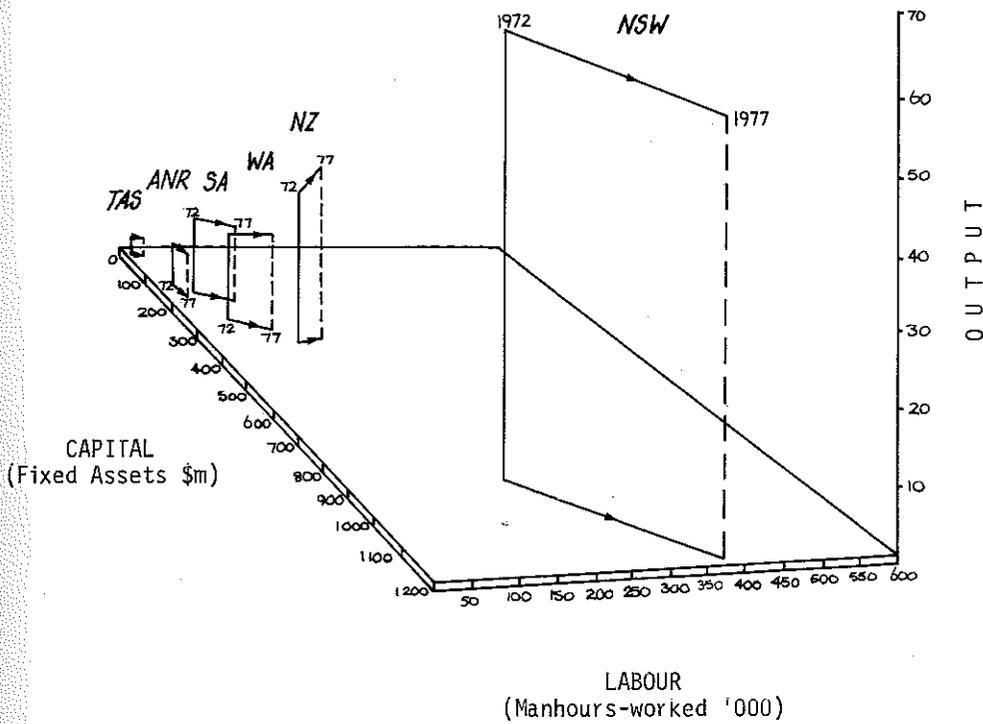
The effect of this calculation, is to scale the ratio of output to capital or to labour by the appropriate elasticity, which is the vital difference between the approach to productivity and the calculation of simple input-output ratios.

In order that these marginal products can be readily interpreted, the marginal products in each case have been multiplied by the average revenue per train-kilometre. This enables one to estimate the marginal product (value) for each railway system. These results are presented in Table 2 for capital and Table 3 for labour. The figures in Table 2 can be interpreted as estimates of the marginal rates of return to capital invested in fixed assets. They reflect the return to capital invested in a range of assets required to expand the total magnitude of railway operations. Results suggest that Westrail was achieving relatively good marginal rates of return to capital while others were not.

Figures in Table 3 indicate whether the marginal dollar spent on labour resulted in a marginal product worth more or less than a dollar. A value of the marginal product less than a dollar suggests that there was excess labour in the railway systems. Results suggest that only Western Australia and Australian National Railways were not operating with excess labour.

FIGURE 1.

AUSTRALIAN RAILWAYS 1972-73 to 1976-77
 TOTAL OUTPUT IN RELATION TO CAPITAL
 ENGAGED AND TOTAL HOURS WORKED.



SOURCE: Railway Annual Reports for New South Wales, New Zealand, Western Australia, South Australia, Australian National Railways and Tasmania for 1972-73 and 1976-77.

RAILWAY PRODUCTION FUNCTIONS

TABLE 2 ESTIMATED MARGINAL PRODUCTIVITY OF CAPITAL EXPRESSED AS THE VALUE OF THE MARGINAL PRODUCT (IN DOLLARS) PER DOLLAR OF FIXED ASSETS: AUSTRALIAN RAILWAY SYSTEMS

Year Ended 30 June	New South Wales	South Australia	Western Australia	Tasmania	Australian National Railways
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Based on Results in Table 1
For Fixed Assets and Wages Plus on-Costs

1973	0.09	0.06	0.07	0.06	0.07
1974	0.08	0.06	0.08	0.05	0.07
1975	0.11	0.10	0.14	0.07	0.10
1976	0.10	0.08	0.15	0.06	0.10
1977	0.06	0.07	0.11	0.04	0.08

Based on Results in Table 1
For Fixed Assets and Wages From Working Expenses

1973	0.09	0.06	0.07	0.06	0.07
1974	0.09	0.07	0.09	0.06	0.08
1975	0.11	0.09	0.14	0.07	0.10
1976	0.14	0.12	0.21	0.08	0.14
1977	0.06	0.06	0.11	0.04	0.07

TABLE 3 ESTIMATED MARGINAL PRODUCTIVITY OF LABOUR EXPRESSED AS THE VALUE OF THE MARGINAL PRODUCT (IN DOLLARS) PER DOLLAR OF WAGES: AUSTRALIAN RAILWAY SYSTEMS

Year Ended 30 June	New South Wales	South Australia	Western Australia	Tasmania	Australian National Railways
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Based on Results in Table 1
For Fixed Assets and Wages Plus On-Costs

1973	0.87	0.68	1.06	0.56	1.15
1974	0.72	0.63	1.05	0.48	1.01
1975	0.60	0.51	0.97	0.37	0.80
1976	0.66	0.53	1.18	0.38	1.10
1977	0.65	0.56	1.18	0.43	1.37

Based on Results in Table 1
For Fixed Assets and Wages from Working Expenses

1973	0.96	0.70	1.15	0.61	1.26
1974	0.79	0.63	1.13	0.50	1.11
1975	0.73	0.59	1.15	0.42	0.09
1976	0.68	0.52	1.18	0.32	1.07
1977	0.73	0.65	1.39	0.48	1.58

TABLE 4

PRODUCTION ELASTICITIES OF LABOUR AND CAPITAL
FOR AUSTRALIAN AND AMERICAN RAILWAYS

Railway System	Production Elasticity With Respect To Labour b_1	Production Elasticity With Respect To Capital b_2	Returns To Scale $b_1 + b_2$	R^2
Australian	0.66 to 0.85	0.23 to 0.64	1.06 to 1.11	.99
<u>American</u>				
241 Southern Railway System	1.07	0.29	1.36	.92
L/N	1.08	0.25	1.33	.92
Seaboard Coast Line	0.87	0.21	1.08	.89
Illinois Central	0.91	0.15	1.06	.73
Gulf, Mobile/Ohio	0.77	0.16	0.93	.60

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CONCLUSIONS

Despite the difficulties associated with the unsatisfactory nature of the data, the study has been able to estimate production elasticities for five separate years 1973, 1974, 1975, 1976 and 1977, of which a summary is presented along with past American studies (Wilson, 1980) for comparison in Table 4. One can see that the Australian results tend to be consistent with those of the U.S., indicating that generally there are increasing returns to scale in the railway industry inferring that there are technical and/or managerial indivisibilities suggesting that there are increasing returns to utilization of rolling stock. This is supported by the results of the value marginal products of capital. Thus one may conclude that even though stock units are inefficiently used, it is still more productive compared with the smaller-scale processes. However, as output grows, management becomes over-burdened and hence less efficient in its role which reflects decreasing returns to scale caused by diminishing returns to management.

One of the difficulties associated with an aggregate cross-section study is that industries face very similar factor price ratios, and it is this problem that contributes to the problem of multicollinearity between the explanatory variables i.e., large railway firms such as New South Wales and Queensland tend to have large quantities of both factors while small firms such as Tasmania and Australian National Railways would have smaller quantities of labour roughly in the same proportion which is attributable to firms facing the same price ratios.

The problem of similar price ratios may be overcome by increasing the size of the sample to include international data on the assumption that the railway industry is on the same production function in each country so that the different ratios of factor prices will generate observations which should trace out the production surface for the railway industry.

Even though the suitability of the data is questionable and the sample size very small, the introduction of slope and intercept shifting dummy variables in the general Cobb-Douglas model has provided five separate cross-section estimates which are statistically reliable.

The unstable nature of the production elasticities may be due to a continual process of adjustment of inputs, but may equally well be due to the inadequacy of the measures on inputs and outputs.

Thus the virtue of the production function is its ability to provide measures of the true factor productivity and returns to scale. If output is only expressed as a simple ratio to labour and to capital respectively, the separately attributable contributions of these two factors to output are not identified. Thus one cannot attribute growth in output to increases in the labour force or investment in capital stock.

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