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Shortest path navigation algorithm for driverless plug-in electric vehicles

Xiang Zhang[1], David Rey[1], Nan Chen[1,2], S. Travis Waller[1]

[1]School of Civil and Environmental Engineering, UNSW Australia, Sydney NSW, 2052, Australia

[2]ARRB, Sydney NSW, 2007, Australia

Email for correspondence: xiang.zhang1@unsw.edu.au

Abstract

Driverless plug-in electric vehicles (DPEVs) have attracted a growing attention in transport engineering due to their low environmental impact and increased level of automation. When compared to regular internal combustion engine vehicles, DPEVs have two characteristics: 1) DPEVs require a higher frequency of recharge operations; 2) DPEVs are controlled by predetermined intelligent algorithms, rather than drivers' experience and judgement. Hence, a critical question is how DPEVs can navigate while minimising travel costs subject to recharge requests. Considering potential delays induced by recharge operations, traditional shortest path algorithms, such as Dijkstra's algorithm, are not completely suitable. In this paper, two predominant recharge modes are considered. One is traditional charging stations, where vehicles must stop and wait to recharge. The other is modern charging lanes, which automatically recharge traversing vehicles. In this case, vehicles might make a detour to catch charging lanes.

This study proposes a shortest path navigation algorithm for DPEVs in traffic systems with recharge facilities. First, a transport network is converted into a fictitious network wherein both charging stations and charging lanes are represented as charging stations with appropriate delay nodes. Second, we introduce a novel mathematical optimisation model for the shortest path navigation problem for DPEVs accounting for recharge delay. Third, we develop a refined label-correcting algorithm accounting for en-route recharge modes and recharge delay for this routing problem. Finally, systematic experiments are conducted to validate the performance of the proposed approach. The results demonstrate that the algorithm can generate route decisions with high computational efficiency. Moreover, we report that the navigation results can be significantly influenced by the number and the location of recharge facilities, recharge time and the distance limits of DPEVs.

1. Introduction and literature review

With the development of our society, more people have their own private vehicles, most of which are internal combustion engine vehicles (ICEVs), particularly gasoline vehicles (GVs). As is well-known, the increasing number of ICEVs has caused a huge amount of petroleum consumption, resulting in serious environmental disruption and energy crisis. By comparison, electric vehicles (EVs), typically plug-in electric vehicles (PEVs) rely primarily or entirely on electricity, which is more economical and environmentally friendly and of greater interest to both planners and users [1]. Besides PEVs, driverless vehicles (DVs) also have received more attentions from both the industry and academia since it helps free up drivers' time, avoid the collisions caused by human errors such as misjudgement and distraction and increase access to vehicles for everyone [2][3]. Therefore, transport-based research efforts associated to PEVs and DVs are experiencing a surge in popularity because of their potential to reduce harmful emissions caused by traditional ICEVs, or a promotion in the intelligent transportation system. Actually, driverless plug-in electric vehicle (DPEV) will definitely attract increasing attentions with a greater emphasis on sustainability and efficiency. As Lawrence D. Burns argued in his paper published in *Nature*, driverless and electric vehicles will revolutionize motoring, which shape the transport future [4]. Previous work has conducted some research in these two sorts of vehicles, but rarely in combination. This study aims to address this gap and make research in shortest path navigation for DPEVs.

One of the major concerns of PEVs lies in the distance limit, which is also called range constraint. Under this restriction, a PEV driver, who always pursues a least travel cost, has to either make a trip with the total distance shorter than the limit, or choose a path or tour along which sufficient recharge facilities are available. This extra task of seeking for recharge sources would unavoidably change the route choice strategy. Furthermore, for a DPEV, it cannot count on a driver that could find a nearby recharge facility empirically. Thus, it is of vital importance to propose a shortest path algorithm to navigate a DPEV.

The shortest path problem is based on the assumption that a traveller will always choose a path with the minimum cost, which is widely acknowledged in the transport research area. However, for a DPEV, the distance limit needs to be addressed, which poses a resource constraint shortest path problem (RCSPP) [5]. Previously, a limited number of studies were conducted in this area. Here, we summarised some of those research outcomes. Handler, G.Y. *et al* made a study on RCSPP in the context of multimode transportation network and developed a Lagrangian relaxation algorithm for problem-solving [6]. Jaffe, J.M addressed RCSPP with both weight and length constraints, and mathematically proved that RCSPP is a NP-complete problem [7]. Erdoğan, S. *et al* formulated a vehicle routing problem with driving range and limited refuelling infrastructures, and developed two construction heuristics to seek for the proposed green route [8]. Jiang, N. *et al* modelled several distance-restrained shortest path problems with range constraints of electric vehicles [9][10]. Chen, N. *et al* used Jiang's outcomes to evaluate the network system performance under various charging station location options [11]. Xie, C. *et al* proposed a shortest path problem with relay requirement and extended the proposed shortest path problem to a traffic assignment problem and explored the impacts of driving range and relay station locations on equilibrium flow patterns [12]. In terms of solution methodologies, Raith, A. *et al.* compared different solution strategies, including standard label-correcting and label-setting methods, a purely enumerative near shortest path approach and the two-phase method, and suggested that the label correcting and setting approaches are preferable in stability [13]. Furthermore, some studies have revealed that label correcting algorithm is the most efficient method. For example, Cabral, E.A. concluded that label correcting algorithm and the network expansion algorithm [14]. Laporte, G. *et al* studied the shortest path problem with replenishment and found that label correcting algorithm is superior to label setting algorithm in terms of computational efficiency [15]. Smith, O.J. *et al* demonstrated the benefits of using label correcting algorithm in aircraft routing problems [16].

However, two major gaps exist in previous studies. First, most of them discarded the recharge time at a charging station in the optimal path decision process, whereas it is an essential part of the total travel cost of an individual trip. Second, previous studies only considered single recharge mode, which is a charging station. But actually, a charging lane is a new type of facility with high recharge efficiency, which was proposed by Highways England [17]. This paper is aimed to address those two gaps, in order to improve and popularise the DPEV shortest path navigation system.

The remainder of this paper is structured as follows. Section 2 includes preparatory work of this study, where an approach of network transformation is proposed. Section 3 formulates the mathematical framework of the DPEV shortest path problem, considering distance limit, two recharge modes, and recharge time. Then, it develops the solution methodology for the proposed shortest path problem. Computational experiments are conducted in Section 4. Finally, conclusions and future work are presented in Section 5.

2. Preparation

2.1. Notations

Notations used throughout the paper are listed as follows unless otherwise are specified.

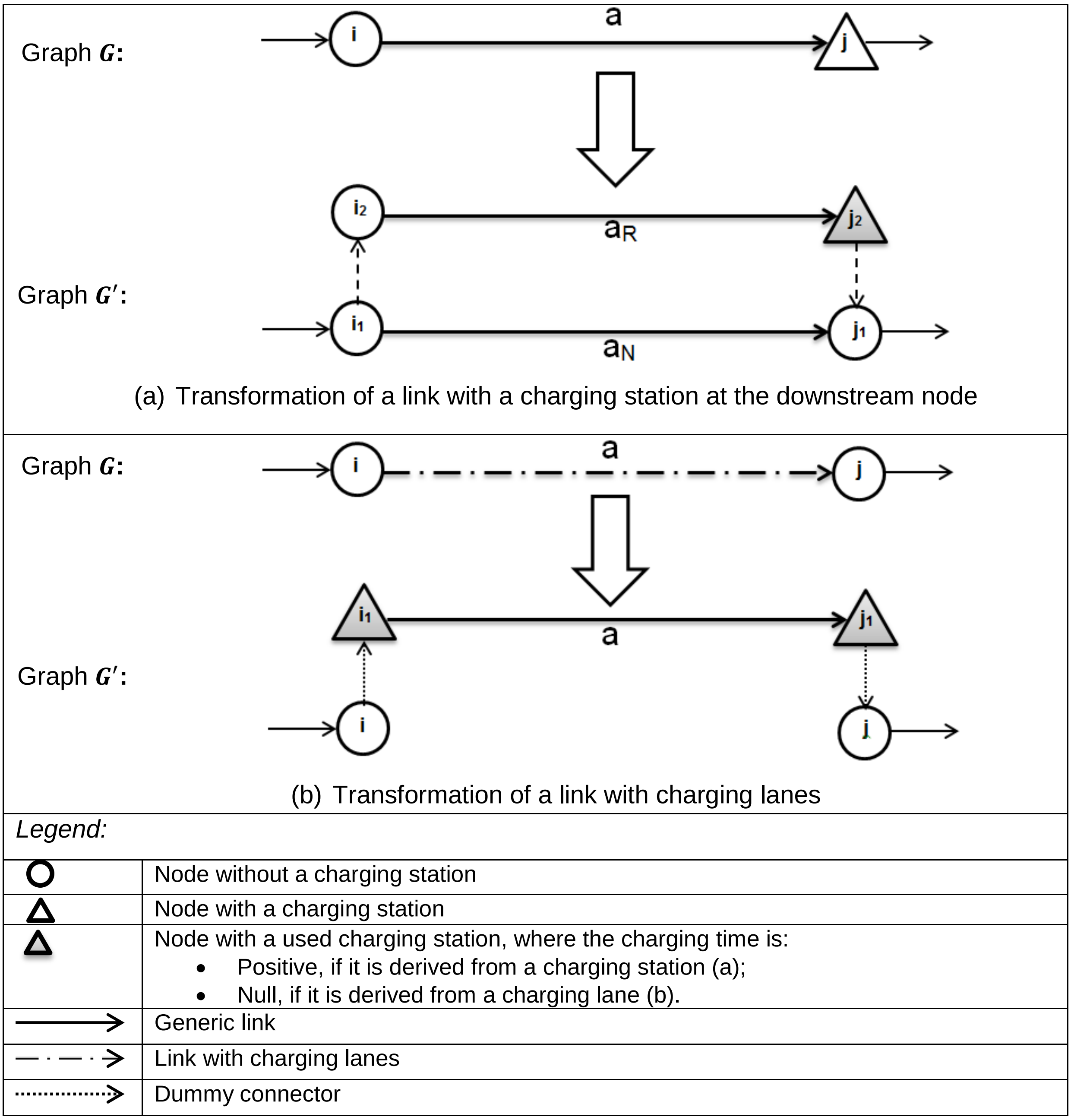
Table 1: Mathematical notations

N	Node set
A	Link set
O	Origin node set
D	Destination node set
Z^2	Origin-Destination (OD) pair set: $Z^2 \subseteq N \times N$
Π	Path set
Π_{rs}	Path set for OD pair (r, s)
Ω	Subpath set
Ω_π	Subpath set for path π
M	Set of classes of vehicles
(i, j)	Link with upstream node $i \in N$ and downstream node $j \in N$
m	Vehicle class index, $m \in M$
t_{ij}	Travel time on link (i, j)
t_{ij}^0	Free flow travel time on link (i, j)
t_π^{rs}	Travel time on path π from origin r to destination s, $\pi \in \Pi_{rs}$
$\delta_{ij,\pi}^{rs}$	Link-path incidence coefficient, which equals one if link (i, j) is on path $\pi \in \Pi_{rs}$, and zero otherwise
pq	Subpath index, which represents a subpath from a used charging station p to the next used charging station q, without any other used charging station on the subpath
$\delta_{ij,\pi}^{rs,pq}$	Link-subpath incidence coefficient, which equals one if link (i, j) is on the subpath pq contained in path $\pi \in \Pi_{rs}$, and zero otherwise
L_m	The maximum distance that the mth class of vehicle can travel without recharge
l_π^{rs}	Distance of path $\pi \in \Pi_{rs}$
$l_\pi^{rs,pq}$	Distance of the subpath pq contained in path $\pi \in \Pi_{rs}$
l_{ij}	Length of link $(i, j) \in A$

2.2. Network transformation

Consider a directed graph $G = (N, A, \Omega)$ where the node set $\boldsymbol{N}$ represents node locations, and the set A and Ω are composed of the directed links (i, j) and subpaths pq respectively in a transport network. Our mathematical formulation of the DPEV shortest path problem works from a transformed network topology, i.e. we introduce virtual nodes and links to represent the decision of using a charging station or lane, as depicted in Figure 1. The key point of the network transformation is to convert both charging stations and charging lanes into used charging stations on a new network.

Figure 1: Network Topology Transformation



(a) Transformation of a link with a charging station at the downstream node

(b) Transformation of a link with charging lanes

A link (i, j) with a charging station at the downstream node shown in Figure 1 (a), can be represented as two fictitious links $a_N = (i_1, j_1)$ and $a_R = (i_2, j_2)$, where the charging station is located at node j_2. Besides, the dummy connectors (i_1, i_2) and (j_2, j_1) have null length and null travel cost. If a DPEV needs to be recharged, then it will choose link a_R to traverse, and a_N otherwise. The lengths of links a_N and a_R are equal:

$$l_{a_N} = l_{a_R} = l_a \tag{1}$$

The travel costs of links a_N and a_R have the following relationship:

$$t_{a_R} = t_{a_N} + \Delta t_j \tag{2}$$

Where Δt_j is the time spent recharging at node j on Graph G, which is assumed to be a positive constant. Given that the travel time is flow-dependent, we have

$$\begin{cases} t_{a_N} = t_{a_N}(x_a) = t_a(x_a) \\ t_{a_R} = t_{a_R}(x_a) = t_a(x_a) + \Delta t_j \end{cases} \tag{3}$$

where

$$\begin{cases} x_a = x_{a_N} + x_{a_R} \\ x_{a_N} = \sum_m x_{a_N,m} \\ x_{a_R} = \sum_m x_{a_R,m} \end{cases} \tag{4}$$

If link $a = (i, j)$ has charging lanes, it can be represented as a link $a = (i_1, j_1)$ with used charging stations located at both endpoints i_1 and j_1, as shown in Figure 1 (b). In this case, the recharge time is zero at both charging stations. Link (i, j) and (i_1, j_1) have the same length and travel cost. The dummy connectors (i, i_1) and (j_1, j) have null length and null travel cost. The significance of the dummy connectors is to avoid the use of charging stations i_1 and j_1 by DPEVs which go through one or both of nodes i and j, but does not traverse the link (i, j).

Based on the transformation above, the network topology G can be transformed into a graph G'. We call G the “realistic graph” and G' the “fictitious graph”. These two graphs are equivalent with regards to link travel costs, recharge service availability and recharge times. Hence, for network system analysis, graph G' is an alternative representation of graph G. Furthermore, one of the strengths of the network topology transformation is to facilitate the formulation of the proposed shortest path problem and traffic assignment problem, which will be demonstrated in the following sections.

3. Methodology

3.1. Model formulation

In this work, we address the shortest path problem with recharge (SPPR) for each class of vehicle among the network. The significant differences between our SPPR and the basic shortest path problem (SPP) are that, in our proposed SPPR:

1) The distance limit of each class of vehicle is modelled;
2) Two recharge modes, charging stations and charging lanes, are considered;
3) Recharge time at a charging station is regarded as a part of travel cost of an individual trip.

In other words, the objective of our proposed SPPR is, over a network $G = (N, A)$, to find the least-cost path between a given OD pair (r, s) considering recharge time, such that the length between any two used consecutive charging stations or between a used charging station and an endpoint of a used charging lane is no greater than the distance limit of the vehicle class. This statement can be simplified with the transformed network: over the transformed network $G' = (N', A')$, the shortest path for each OD pair is chosen, wherein each subpath pq connecting two consecutive charging stations must satisfy the distance constraint. Based on the latter, we formulate the proposed SPPR. Let $\Omega_\pi{}'$ be the set of subpath pq contained in a path π connecting OD pair (r, s). Link variable τ_{ij} is defined as follows: $\tau_{ij} = 1$ if link (i, j) is contained in the chosen path, and $\tau_{ij} = 0$ otherwise. Similarly, a subpath-link variable $\tau_{ij}^{pq} = 1$ indicates that link (i, j) lies on the subpath pq and $\tau_{ij}^{pq} = 0$ otherwise. We assume that each vehicle starts its trip with full charge, or the adjusted (reduced) driving range L_m is known otherwise. Another assumption is that all the vehicles that traverse a charging lane or use a charging station will be fully recharged. The mathematical formulation of the proposed SPPR can be written as follows:

P, $SPPR(m, r, s)$

$$\min_{\pi} t_\pi = \sum_{(i,j)\in A'} t_{ij} \cdot \tau_{ij} = \sum_{pq\in \Omega_\pi'} \sum_{(i,j)\in A'} t_{ij} \cdot \tau_{ij}^{pq} \tag{5}$$

subject to

$$\sum_{j\in N'} \tau_{ij} - \sum_{k\in N'} \tau_{ki} = \begin{cases} 1 & if\ i = r \\ -1 & if\ i = s \\ 0 & Otherwise \end{cases} \tag{6}$$

$$\sum_{j\in N'} \tau_{ij}^{pq} - \sum_{k\in N'} \tau_{ki}^{pq} = \begin{cases} 1 & if\ i = p \\ -1 & if\ i = q \\ 0 & Otherwise \end{cases} \tag{7}$$

$$\sum_{(i,j)\in A'} l_{ij} \cdot \tau_{ij}^{pq} \le L_m, \forall pq \in \Omega_\pi' \tag{8}$$

$$\tau_{ij} \in \{0,1\}\ \forall (i,j) \in A' \tag{9}$$

$$\tau_{ij}^{pq} \in \{0,1\}\ \forall (i,j) \in A', \forall pq \in \Omega_\pi' \tag{10}$$

The objective function (5) is to minimise the total travel cost of an individual traveller. Constraint (6) is the flow conservation constraint, sending a unit flow through a path connecting OD pair (r, s). The objective function (5) and (6) have the same format as those in the basic SPP model, whereas, in comparison with the basic SPP model, the objective and constraints in *P1* are established upon the transformed network G' instead of the original network G. Another improvement with respect to the basic SPP model lies in the extra constraints (7) and (8), which respectively impose flow conservation constraints for each subpath pq and subpath distance constraints for each vehicle class. Constraints (9) and (10) indicate the domain of the link variables τ_{ij} and τ_{ij}^{pq}.

The above SPPR is framed as a resource constrained shortest path problem (RCSPP). In fact, no polynomial time algorithm is likely to be developed for solving RCSPP since it is known to be NP-complete [18][19]. The solution approach of our proposed SPPR will be discussed in the next section.

3.2. Solution approach

Using the transformation described in Section 2.2, we get a fictitious graph G', where a DPEV will be fully recharged at all charging stations along its selected route. In addition, network G' accounts for recharging time through link travel costs. Therefore, solving the proposed shortest path problem over the derived network G', doesn't require considering charging lanes or recharging time at stations. For this sort of SPPR without those two considerations, a pseudo-polynomial time solution algorithm has been proposed by Jiang *et al.* [20]. Their approach is mainly a modified label-correcting algorithm. In this paper, we propose to refine this solution algorithm and solve the proposed SPPR within the realistic network $G = (N, A)$.

First, we define a label set $L_i = \{(t_i, l_i)_k = (t_i^k, l_i^k)\}$ for each node i. For each label $(t_i, l_i)_k$ in the set L_i: t_i represents the travel cost (time) from the origin node to node i, which includes the time spent on recharge at all used charging stations; l_i represents the distance of the subpath from the latest used charging station or charging lane to node i, where the subpath is on the path from origin node to node i; k is the label index. We denote the set of indices of the treated permanent labels for node i as P_i, and the set of indices of the untreated temporary labels for node i as T_i.

Second, we say that a label is feasible for the mth class of vehicle if $l_i^k \le L_m$. Consider that more than one feasible label is set to node i, including two of them denoted as (t_i^k, l_i^k) and (t_i^w, l_i^w). The label (t_i^k, l_i^k) is said to be dominated by the label (t_i^w, l_i^w), if $t_i^k > t_i^w$ and $l_i^k \ge l_i^w$, or $t_i^k \geq t_i^w$ and $l_i^k > l_i^w$. With this definition, we assume that all the travellers would prefer the route with both less travel cost and less distance with respect to the latest recharge. Furthermore, we say that, for the mth class of vehicle, a label (t_i, l_i) is efficient if it is not dominated by any other label at node i. Besides, we denote the set of nodes with charging stations as N_{cs}, where $N_{cs} \subseteq N$; the set of links with charging lanes as A_{cl}, where $A_{cl} \subseteq A$. With the definitions above, the procedure of the refined label-correcting algorithm is summarized below.

Step 1: Initialization

- Set the label set of the origin node r as $L_r = \{(0,0)_1\}$ and the label set of any other node as $L_i = \{(\infty, \infty)_1\},\ \forall i \in N \backslash \{r\}$.
- Set the index sets: $T_r = \{1\}$; $T_i = \emptyset, \forall i \in N \backslash r$; $P_i = \emptyset, \forall i \in N$.

Step 2: Select the critical temporary label

- Select the critical label $\left(t_i^k, l_i^k\right)$, $i \in N, k \in T_i$, such that l_i^k is the minimal for all the untreated labels.
- Update T_i and P_i: $T_i = T_i \backslash k$, $P_i = P_i \cup \{k\}$.

Step 3: Generate new labels

For each link $(i,j) \in A$, if $l_i^k + l_{ij} < L_m$:

- If $(i,j) \in A_{cl}$, the new generated label $\left(t_j^\gamma, l_j^\gamma\right)$ for node j is $\left(t_i^k + t_{ij}, 0\right)$;
- If $j \in N_{cs}$, the new generated labels $\left(t_j^\gamma, l_j^\gamma\right)$ for node j are $\left(t_i^k + t_{ij} + \Delta t, 0\right)$ and $\left(t_i^k + t_{ij}, l_i^k + l_{ij}\right)$;
- If $(i,j) \notin A_{cl}$ and $j \notin N_{cs}$, the new generated label $\left(t_j^\gamma, l_j^\gamma\right)$ for node j is $\left(t_i^k + t_{ij}, l_i^k + l_{ij}\right)$.

Step 3: Update label sets and label index sets

- If $\left(t_j^\gamma, l_j^\gamma\right)$ is not dominated by any $\left(t_j^w, l_j^w\right) \in L_j$, $w \in T_j \cup P_j$, then $L_j = L_j \cup \left(t_j^\gamma, l_j^\gamma\right)$ and $T_j = T_j \cup \gamma$;
- If any $\left(t_j^w, l_j^w\right)$, $w \in T_j$, is dominated by $\left(t_j^\gamma, l_j^\gamma\right)$, then $L_j = L_j \backslash \left(t_j^w, l_j^w\right)$ and $T_j = T_j \backslash w$.

Step 4 Termination Criteria

- If $T_i = \emptyset, \forall i \in N$, then stop. Otherwise, return to *Step 2*.

4. Case analysis

The purpose of this case analysis is: 1) to demonstrate the validity of the proposed approaches; 2) to examine the impacts on optimal route choice decision by different factors, including recharge facility availability, distance limit and recharge time. We use the well-known Sioux-Falls (SF) network to conduct the computational experiments. The network data can be obtained online [21]. The models and algorithms are implemented in Python. The tests are conducted on a Windows 7 platform with an Intel Core i7-4770 processor at 2.41 GHz and 2.0 Gb of RAM.

In this section, we solve the proposed SPPR by using the proposed refined label-correcting algorithm. The link travel time under the free flow pattern is used as a proxy. Experiments numbered I - IV are conducted, each of which varies one of the four following factors respectively: charging station allocation, distance limit, recharge time and charging lane allocation. Four O-D pairs are randomly selected. The results are summarised in Tables 2 - 5.

The average run time of each implemented shortest path is around 0.2 second, which underlines the computational efficiency.

Table 2: Computational results of Experiment I

Scenario Description				
Scenario	Set of Nodes with Charging Stations N_R	Recharge Time at Charging Stations	Set of Links with Charging Lanes	Distance Limit
(i)	{2,5,7,11,13}	$\Delta t_3 = 5, \forall i \in N_R$	{∅}	9
(ii)	{2,5,7,11,13,17}	$\Delta t_3 = 5, \forall i \in N_R$	{∅}	9
(iii)	{2,3,5,7,11,13,17}	$\Delta t_3 = 5, \forall i \in N_R$	{∅}	9
(iv)	{2,3,5,7,8,11,13,17}	$\Delta t_3 = 5, \forall i \in N_R$	{∅}	9

Shortest Path								
OD	Scenario							
	(i)		II		(iii)		(iv)	
	SP[a]	t_{SP}[b]	SP	t_{SP}	SP	t_{SP}	SP	t_{SP}
(1,20)	1-2-6-5-6-8-7-18-20	45	1-2-6-5-6-8-7-18-20	45	1-3-4-5-6-8-7-18-20	40	1-2-6-8-7-18-20	32
(1,22)	1-2-6-5-4-11-12-13-24-21-22	61	1-2-6-5-6-8-7-18-16-17-19-15-22	59	1-3-12-13-24-21-22	30	1-3-12-13-24-21-22	30
(2,20)	2-6-5-6-8-7-18-20	34	2-6-5-6-8-7-18-20	34	2-6-5-6-8-7-18-20	34	2-6-8-7-18-20	21
(2,22)	2-6-5-4-11-12-13-24-21-22	50	2-6-5-6-8-7-18-16-17-19-15-22	48	2-6-5-4-3-12-13-24-21-22	46	2-6-8-16-17-19-15-22	32

a. SP represents the shortest path.
b. t_{SP} is the shortest path travel time, including recharge time at a charging station.

From the results of Experiment I, we can see that the shortest paths are altered in some cases due to the change in the charging station allocation. Besides, the shortest path may contain a cycle because of the need to recharge, *e.g.* Scenario II. Another expected outcome is that the shortest path travel time tends to decrease with the increasing supply of charging stations.

Table 3: Computational results of Experiment II

Scenario Description				
Scenario	Set of Nodes with Charging Stations N_R	Recharge Time at Charging Stations	Set of Links with Charging Lanes	Distance Limit
(i)	{2,5,7,11,13,17}	$\Delta t_3 = 5, \forall i \in N_R$	{∅}	9
(ii)	{2,5,7,11,13,17}	$\Delta t_3 = 5, \forall i \in N_R$	{∅}	10
(iii)	{2,5,7,11,13,17}	$\Delta t_3 = 5, \forall i \in N_R$	{∅}	15
(iv)	{2,5,7,11,13,17}	$\Delta t_3 = 5, \forall i \in N_R$	{∅}	20

Shortest Path								
OD	Scenario							
	(i)		II		(iii)		(iv)	
	SP	t_{SP}	SP	t_{SP}	SP	t_{SP}	SP	t_{SP}
(1,20)	1-2-6-5-6-8-7-18-20	45	1-2-6-8-7-18-20	32	1-3-12-13-24-21-20	29	1-2-6-8-7-18-20	27
(1,22)	1-2-6-5-6-8-7-18-16-17-19-15-22	59	1-2-6-8-7-18-16-17-19-15-22	46	1-3-12-13-24-21-22	25	1-3-12-13-24-21-22	20
(2,20)	2-6-5-6-8-7-18-20	34	2-6-8-7-18-20	21	2-6-8-7-18-20	21	2-6-8-7-18-20	16

(2,22)	2-6-5-6-8-7-18-16-17-19-15-22	48	2-6-8-7-18-16-17-19-15-22	35	2-6-8-7-18-20-22	26	2-6-8-7-18-20-22	26

From the results of Experiment II, we find that the greater the distance limit is, the least travel time an electric vehicle can bear. This can be explained as follows: First, a longer distance limit leads to a looser range constraint, which means that a vehicle might not necessarily do a detour for recharge requirement, reducing the total travel time. This can be seen for example, by comparing Scenarios (i) and (ii). A second reason is that even though, sometimes, a larger distance limit does not have an influence on optimal route choice, a vehicle is not in need of recharge at every charging station along its travelled path, resulting in recharge time savings. Such situation can be found, for instance, by comparing Scenarios (iii) and (iv) with OD pairs (1,22) and (2,20).

Table 4: Computational Results of Experiment III

Scenario Description

Scenario	Set of Nodes with Charging Stations N_R	Recharge Time at Charging Stations	Set of Links with Charging Lanes	Distance Limit
(i)	{2,3,5,7,8,11,13,17}	$\Delta t_3 = 15;$ $\Delta t_i = 1, \forall i \in N_R \backslash 3$	$\{\emptyset\}$	9
(ii)	{2,3,5,7,8,11,13,17}	$\Delta t_8 = 15;$ $\Delta t_i = 1, \forall i \in N_R \backslash 8$	$\{\emptyset\}$	9
(iii)	{2,3,5,7,8,11,13,17}	$\Delta t_{17} = 15;$ $\Delta t_i = 1, \forall i \in N_R \backslash 17$	$\{\emptyset\}$	9
(iv)	{2,3,5,7,8,11,13,17}	$\Delta t_j = 15, j = 3,8,17;$ $\Delta t_i = 1, \forall i \in N_R \backslash \{j\}$	$\{\emptyset\}$	9

Shortest Path

OD	Scenario							
	(i)		II		(iii)		(iv)	
	SP	t_{SP}	SP	t_{SP}	SP	t_{SP}	SP	t_{SP}
(1,20)	1-2-6-8-7-18-20	24	1-3-4-5-6-8-7-18-20	28	1-2-6-8-7-18-20	24	1-2-6-5-6-8-7-18-20	33
(1,22)	1-2-6-8-16-17-19-15-22	31	1-3-12-13-24-21-22	22	1-3-12-13-24-21-22	22	1-3-12-13-24-21-22	36
(2,20)	2-6-8-7-18-20	17	2-6-5-6-8-7-18-20	26	2-6-8-7-18-20	17	2-6-5-6-8-7-18-20	26
(2,22)	2-6-8-16-17-19-15-22	24	2-6-5-4-3-12-13-24-21-22	34	2-6-5-4-3-12-13-24-21-22	34	2-6-5-4-11-12-13-24-21-22	38

Results of Experiment III reveal the impacts of different recharge time on the route choice strategies and travel time. We can find that when recharge time increases, a vehicle might make a detour or cycle on top of the previous optimal path for recharge purpose. For example, in Scenario (iv), recharge time at Node 8 and 17 is increased as compared to Scenario (i). As a result, a cycle 6-5-6 is added to the optimal paths connecting OD pairs (1,20) and (2,20).

Table 5: Computational Results of Experiment IV

Scenario Description

Scenario	Set of Nodes with Charging Stations N_R	Recharge Time at Charging Stations	Set of Links with Charging Lanes	Distance Limit
(i)	{2,5,7,11,13,17}	$\Delta t_3 = 5, \forall i \in N_R$	$\{\emptyset\}$	9
(ii)	{2,5,7,11,13,17}	$\Delta t_3 = 5, \forall i \in N_R$	{(6,8)}	9
(iii)	{2,5,7,11,13,17}	$\Delta t_3 = 5, \forall i \in N_R$	{(10,15)}	9

(iv)	{2,5,7,11,13,17}		$\Delta t_3 = 5, \forall i \in N_R$		{(6,8),(10,15)}		9	
Shortest Path								
OD	Set of Links with Charging Lanes							
	(i)		II		(iii)		(iv)	
	SP	t_{SP}	SP	t_{SP}	SP	t_{SP}	SP	t_{SP}
(1,20)	1-2-6-5-6-8-7-18-20	45	1-2-6-8-7-18-20	27	1-2-6-5-6-8-7-18-20	45	1-2-6-8-7-18-20	27
(1,22)	1-2-6-5-6-8-7-18-16-17-19-15-22	59	1-2-6-8-16-17-19-15-22	38	1-2-6-5-9-10-15-22	42	1-2-6-8-16-10-15-22	36
(2,20)	2-6-5-6-8-7-18-20	34	2-6-8-7-18-20	16	2-6-5-6-8-7-18-20	34	2-6-8-7-18-20	16
(2,22)	2-6-5-6-8-7-18-16-17-19-15-22	48	2-6-8-16-17-19-15-22	27	2-6-5-9-10-15-22	31	2-6-8-16-10-15-22	25

In Experiment IV, we equip some of the links with charging lanes. We can see that having more charging lanes will likely decrease shortest path travel times. For example, with links (6,8) and (10,15) equipped with charging lanes, the shortest path travel time for each OD pair is reduced by 40.0%, 39.0%, 52.9% and 47.9% respectively, as compared to Scenario (i). On the other hand, in some cases, charging lanes make no difference to route choice decision, which can be indicated, for example, by comparison of Scenarios (ii) and (iv) for OD pairs (1,20) and (2,22).

5. Conclusions and future work

This study addresses the shortest path navigation problem of driverless plug-in electric vehicles. It formulates the mathematical framework of resource constraint shortest path problem. As compared to previous studies, two key improvements are made: 1) recharge time is taken as a part of individual trip cost in the optimal path decision process; 2) two charging modes are available among the network, *i.e.* charging stations and charging lanes. Then, the modified label correcting algorithm is developed to solve the proposed problem. Finally, case study is conducted based on the middle-sized Sioux-Falls Network. The result provides insights into the complicated behaviour of the proposed problem. It demonstrates that four factors have significant impacts on the optimal route decision process, which are charging station allocation, charging lane allocation, recharge time and distance limit. Specifically, the whole travel cost of an individual trip can be reduced by increasing the distance limit or the amount of charging services, or decreasing recharge time. Furthermore, different charging facility locations can lead to different route choice strategies and travel costs.

The outcome of this work can be applied in the navigation system of driverless plug-in electric vehicles. For future research directions, the proposed model and algorithm can be extended to the traffic assignment problems with the driving range. Meanwhile, we can investigate how both driverless internal combustion engine vehicles and plug-in electric vehicles can be accommodated under user equilibrium conditions. After that, systematic network evaluation and project rankings can be conducted, wherein economic analysis is of particular interest in this context. From this work, we can analyse the range-constrained route choice behaviour and the savings in travel costs in response to the updates on recharge facility allocations which are associated with the construction expenditure and budgets. In other words, given the real-world economic data, we can estimate benefit-cost ratio for each potentially selected planning policy based on this work, which will technically support the government’s decision-making process. Last but not least, we expect that the modelling and solution methodologies presented in this work would potentially intrigue interest of a broader transport field including multimodal transport route choice and freight network optimisation.

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